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Graphical Methods for Depicting Combat Units

by

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Analysis of battlefield training is being enhanced with computer screens that display maps of the battlefield. At the National Training Center (NTC), Fort Irwin, CA, training battles are digitally recorded to be analyzed later by military officers, tacticians and analysts. A large screen computer generated map display is used to unfold the battle one piece at a time to see what caused the forces to achieve success or failure. As an improvement to current battlefield displays, graphical methods and techniques are presented for aggregating the elements of combat units. The techniques focus on locating a unit, graphically summarizing the movement of a unit, and graphically depicting a unit's dispersion on the battlefield. These methods are intended for use in post-battle analysis software platforms like the ones used at the NTC.						
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ABSTRACT

Analysis of battlefield training is being enhanced with computer screens that display maps of the battlefield. At the National Training Center (NTC), Fort Irwin, CA, training battles are digitally recorded to be analyzed later by military officers, tacticians and analysts. A large screen computer generated map display is used to unfold the battle one piece at a time to see what caused the forces to achieve success or failure. As an improvement to current battlefield displays, graphical methods and techniques are presented for aggregating the elements of combat units. The techniques focus on locating a unit, graphically summarizing the movement of a unit, and graphically depicting a unit's dispersion on the battlefield. These methods are intended for use in post-battle analysis software platforms like the ones used at the NTC.

THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

ACKNOWLEDGEMENT

The programming language *APL 2* and the interactive software system *A Graphical Statistical System* (AGSS) are used at the Naval Postgraduate School under a test site agreement with IBM Research. We are indebted to Dr. Peter Welch for making this possible.

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I. INTRODUCTION

A. PURPOSE

The computer screen is frequently used in the military to show a visual display of the battlefield. Technological advances in computing power and speed have increasingly put more demands on the computer to assist man in preparing for war. The most common applications are seen in combat modeling and simulations. During the last decade, the computer has also been used as a tool for analyzing battlefield performance. Today, Army commanders, who have the battlefield experience and tactical expertise to evaluate battlefield performance, are using computer generated images to evaluate tactical performance.

At the National Training Center (NTC) at Fort Irwin, CA, computers are used to record training battles so that the battles can be "replayed" for analysis. After each battle, there is an after-action review (AAR) with the key leaders from the red and blue forces discussing the good and bad events in the battle. In the AAR, a computer display is used to reenact the battle. This display resembles a modern, high resolution computer battle simulation. It has a terrain map overlaid with up to five hundred red and blue squares, each plotted at the location of an enemy or friendly weapon system. As the battle is reenacted, the symbols move on the display to replicate their actual movement during the battle. Generally, as each crucial event of the battle is discussed in the AAR, the visual display is paused. The display becomes a static picture of the symbols with all perceived movement removed. To help the viewers make sense of the static symbols and visualize the role of each unit in the battle, hand drawn circles that group the weapons together by unit are frequently placed on the computer display.

Figure 1 is a map display similar to those used at the NTC, without the terrain features and colorings. It represents a 10 kilometer by 20 kilometer Universal Transverse Mercator (UTM)¹ mapping of 50 red vehicles (open box) and 100 blue vehicles (solid box). One can see that there is a significant amount of symbol over-plotting at this scale. No organizational details are depicted other than differentiation between friend and enemy. It is apparent that there is a limited amount of information available from a display of this type. To further demonstrate the limits of this type of display, Figure 2 "zooms-in" on the blue forces shown in Figure 1. The 60% increase in scale does not reveal much more about the battlefield situation, and over-plotting is still evident. Seeing this, one asks: "Is there a better way to display battle information that would enhance the analysis of the battle?"

One alternative would be to reduce the resolution of the display and show the *combat units* in a manner that is easily understood. Using something like the evaluator's hand drawn circles or some other symbol to represent each unit during the replay, one could view the actions of the *units* fighting the battle. Since these displays are generated from a computer database, it is possible to sketch the movement path for each unit. Figure 3 is an example of the results of this thesis showing two units moving on the battlefield. A visualization of this type may lead to better tactical analysis of the commander's battle plan and the plan's execution. The purpose of this thesis is to develop aggregated symbols and methods for using them with data collected at the NTC, to show a unit's location, its speed and direction of movement, and its degree of dispersion.

¹The UTM map is a military standard map developed by the U.S. Army and adopted by NATO. [Ref. 1:p.200]

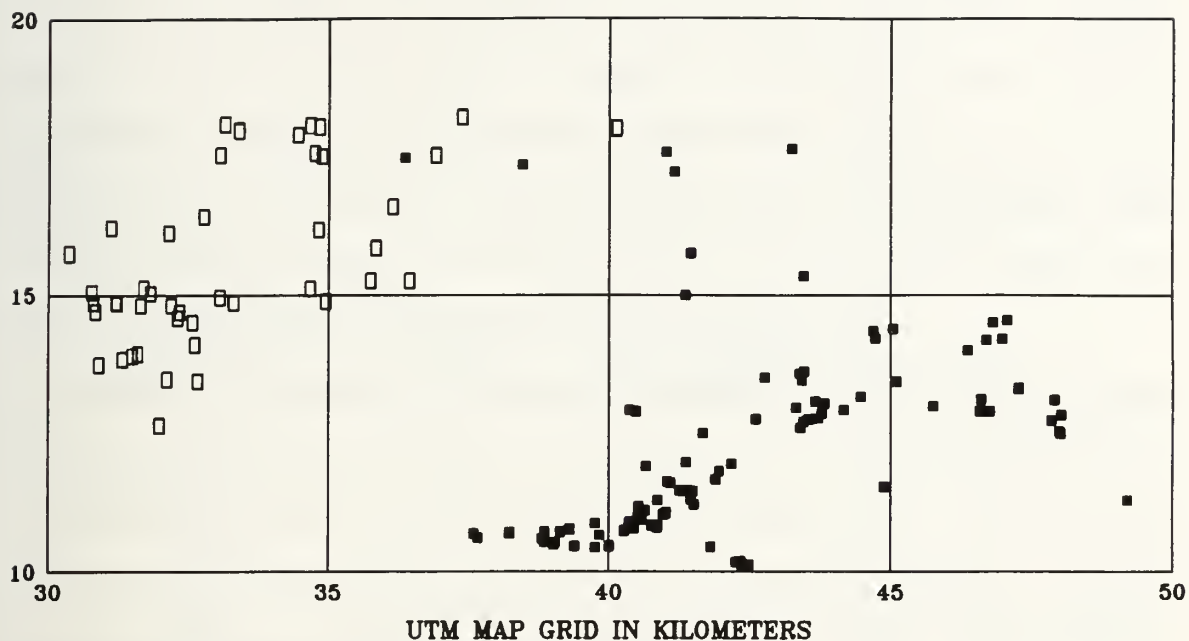


Figure 1. Example of a Battlefield Map Display

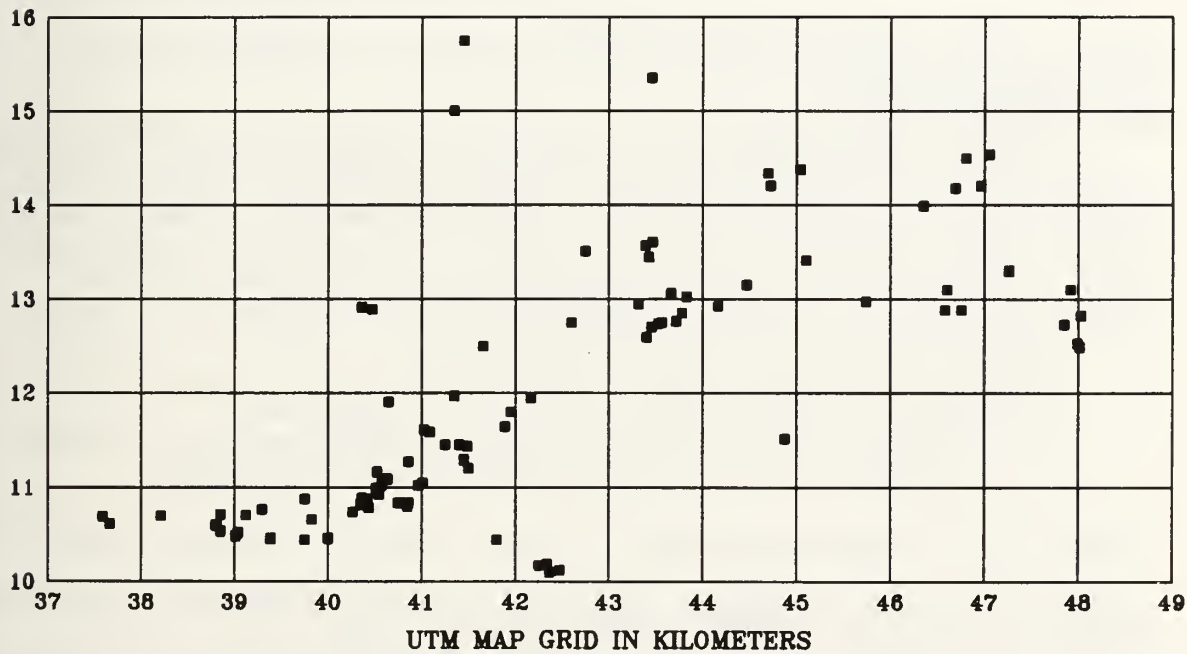


Figure 2. Example of a Battlefield Map Display at a Larger Scale

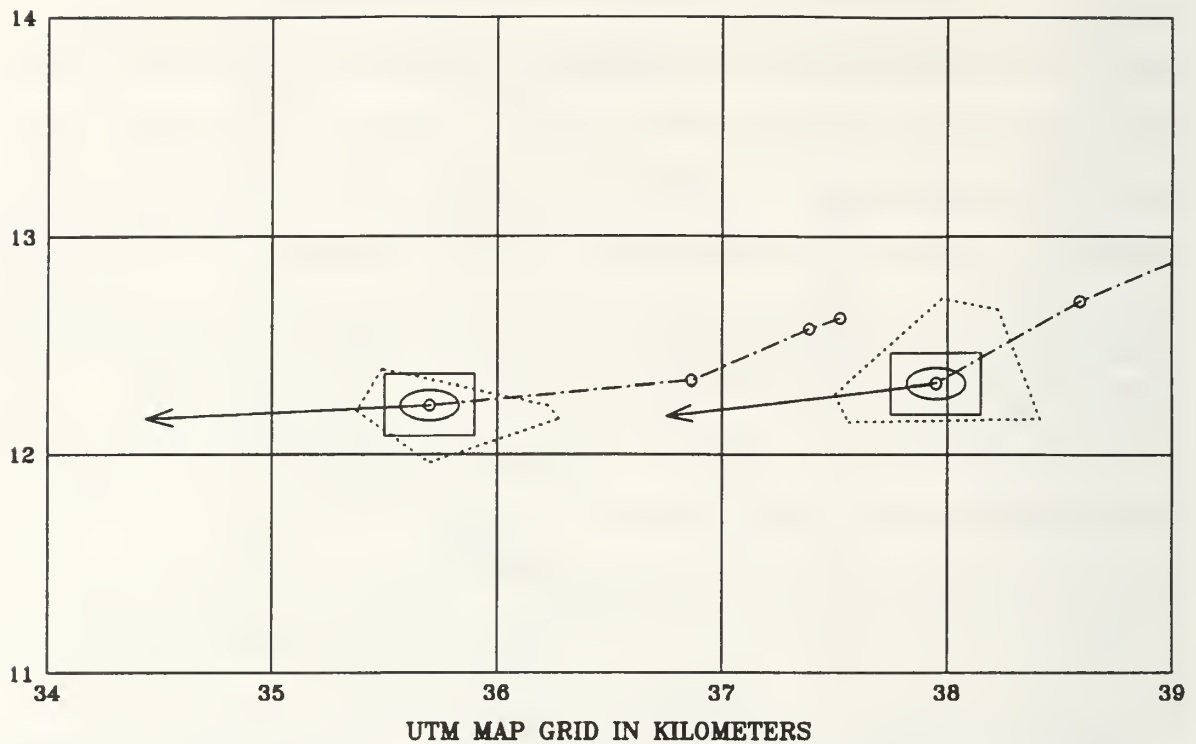


Figure 3. Example of Map Display with Aggregated Unit Maneuver Graphics

B. BACKGROUND

The United States Army has sought during the last decade to improve battle readiness by training combat units under realistic battlefield conditions. To do this the Army has invested heavily in three Combat Training Centers (CTC) around the world:

- National Training Center (NTC) at Fort Irwin, CA
- Joint Readiness Training Center (JRTC) at Fort Chaffee, AK
- Combat Maneuver Training Center (CMTTC) at Hohenfels, Germany.

At the CTC's, Army units participate in realistic, combat-like training against a well-trained enemy force. A group of observer-controllers (O/C) control the exercises and evaluate the training. After each battle, O/C's provide immediate feedback to the commanders and their staffs on their unit's performance.

Since its conception, the NTC was designed to collect data for objective assessment of training effectiveness. Instrumentation was developed to track player locations, player status (dead or alive), and player engagements. Sophisticated equipment such as the Multiple Integrated Laser Engagement System (MILES) replicates actual weapon firings and "kills" with safe laser beams and laser sensors. The data captured, along with O/C evaluations, form the basis for post-battle analysis. Current analysis at the NTC is strictly focused on attrition based measures of performance.

In 1986, the Government Accounting Office reported that the United States Army was not realizing the NTC's potential. The comments were primarily directed at the lack of analytical direction used to design the data collection efforts. From the beginning, data collection has focused on recording the battles; little consideration was given to specific data requirements, nor to what the data would be used for. Since the 1986 report, the Army has sought to redefine its efforts by developing objective means for measuring combat training performance.

The Army's TRADOC¹ Analysis Command (TRAC) specifically works to improve the assessment of unit performance. TRAC is the proponent of a study called Battle Enhanced Analysis Methodologies (BEAM). This study seeks to go beyond attrition based measures of performance by analyzing battlefield performance from a doctrinal viewpoint. This will form a basis for quantifying difficult elements such as tactical employment and leadership. The assessment of doctrinal effectiveness should provide measures of performance that are easily understood by Army tacticians.

¹TRAC's parent organization, TRADOC, is the Army's Training and Doctrine Command

Current principles for employing U.S. Army forces are called the Airland Battle Doctrine and are evolving into what is called AirLand Operations. The current Airland Battle Doctrine has four basic tenets that are considered essential to decisive battlefield operations. The four tenets are:

- Initiative--a willingness and ability to act independently within the higher commander's intent.
- Agility--the ability of friendly forces to act faster than the enemy.
- Depth--engaging the enemy with fires and attacks on his flanks, rear, and support echelons.
- Synchronization--the arrangement of battlefield activities in time, space and purpose to produce maximum relative combat power at the decisive point. [Ref. 2:pp. 14-18]

Clearly, these tenets are complex, with many attributes which can be measured or quantified in various ways. The BEAM project seeks to find the best quantifications of these tenets for use in training.

At this time, several projects in the BEAM study are concentrating on measures associated with synchronization, developing computer displays to pictorially show various measures of combat power on the battlefield. These measures are based upon the characteristics of the individual weapons and the line of sight from the position of each weapon. The overlap of these effects, or the cumulative effects of all the weapons show concentrations of combat power on the battlefield. [Ref. 3]

This thesis compliments the other projects in the BEAM study. It focuses on the graphic representation of movement, an integral part of all four tenets of Airland Battle Doctrine.

C. PROBLEM DESCRIPTION AND SCOPE

The displays in this study were derived from training battles fought at the NTC that were classified as "deliberate attacks." These offensive battles, which are characterized

by speed and maneuver, were sought because the aim was to visually display a unit's movement. The deliberate attack is an offensive mission that begins with thorough planning and preparation so that it is fully synchronized. That is, weapon fires and movement on the battlefield are integrated to concentrate the maximum shock and violence against the enemy. When initiated, the friendly forces move to engage the enemy, attacking with every available asset to seize their objective. [Ref. 2:pp. 96-99]

While aggregation can be used at almost any level, the size of a unit depicted with a symbol usually depends upon the echelon of command (company, battalion, brigade, or higher¹) using the symbols. Since the NTC was designed for training battalion sized units, the participants in NTC battles consist of a battalion and its attachments. Therefore, this thesis focuses on the major subordinate unit in a battalion--the company. A typical armor company has 14 tanks which will be aggregated in this thesis for study as a single unit.

1. The Location and Movement of a Unit

A purpose of this thesis is to develop the use of aggregated symbols to show a unit's location. To replace the symbols for the 14 tanks in a company with one symbol, one must first decide where on the map that symbol should be located. Instinctively, it should be located at the center of the unit, but there are many definitions of center. The 14 tanks in a company may occupy from as little as 100 square meters (parked side by side and bumper to bumper) to as much as three square kilometers or more when maneuvering on the battlefield. Moreover, they may be unevenly dispersed throughout the area with a large concentration on one side and a few "stragglers" expanding the area on another side. These stragglers, (often called outliers) can greatly affect some

¹Brigades have two or more battalions. Battalions have two or more companies. Companies have two or more platoons.

popular measures of the center, pulling it away from the "operational" or "combat" center of the company. This thesis seeks a measure of the center of a unit that reflects the locus of a unit's influence.

Once a good method for locating a unit's center is established, the usefulness of that center can be explored. For example, displaying the center of a unit at different points in time will yield a graphical plot of the unit's maneuver in terms of its speed and direction of movement. Locating the center of the unit and displaying movement is developed in Chapter II.

2. The Dispersion of a Unit

The remaining part of the problem is to meaningfully display the dispersion of the weapons in a unit. A simple graphic measure is sought that reflects the dispersion of the unit, while preserving most of the information that would be available from seeing each weapon plotted separately on the map. One disadvantage of an aggregated symbol is that it is only a summary of the positions of the weapons in the unit. Without the visual impression given by the two dimensional plot of the weapons in a unit, the information about the dispersion of the unit is hidden. In this respect, a simple hand sketch of the unit's perimeter would be useful. It can show the relative distance between a unit's weapons and reveal the manner in which the unit is spatially arrayed on battlefield. Chapters III and IV cover the dispersion of a unit.

D. DATA COLLECTION

Battlefield data used in this thesis was collected from the archives at the Army Research Institute, Presidio, Monterey, CA (ARI-POM). The training battles examined were conducted at the NTC between October 1989 and May 1990. The data was originally collected from the instruments at the NTC and transferred to a digital relational database on a VAX computer. Using INGRESS, a database management system, one

can make queries to access the data. Mr. Rick Crenshaw, the Database Manager, assisted in accessing the data by writing the queries used here.

To review the selected deliberate attack battles, a battle playback program, GNATT II (General-Purpose NTC Analysis of Training Tool, second version), developed by Mr. Crenshaw for the MS-DOS family of computers, was used. Five separate data files for each battle were captured from the VAX computer in the format required for GNATT II:

- PL.DAT Player position location data
- ENG.DAT Engagement positions
- ORG.DAT Organization list
- CMT.DAT Control measure data
- MORTALTY.DAT Weapon system status.

These files, specifically, PL.DAT, ORG.DAT, and MORTALTY.DAT, contain the raw data used in this thesis. Appendix B contains sample extracts of the data files.

The battle data files were then imported into A Graphical Statistical System (AGSS), a statistics software package developed by IBM for the MS-DOS family of computers as well as AIX workstations and mainframes. All data manipulations and graphical presentations were done using AGSS.

E. CHARACTERISTICS OF NATIONAL TRAINING CENTER DATA

The plan for instrumentation and data collection at the NTC included equipment that was state of the art when the NTC was designed. Since then, technological advances have far out paced improvements made at the NTC. Still, the Army has aggressively pursued modernization within budgetary constraints. For example, a SUN computer workstation network recently replaced the outdated computer communication links throughout Ft. Irwin. This has greatly enhanced speed and data storage

capabilities. Perhaps the next major enhancement will incorporate the global positioning system to more accurately locate players on the battlefield.

The data collected has several known anomalies. Some of these anomalies were discovered during research for this thesis and later verified with Mr. Jack Baldwin, BDM Corporation. These anomalies come from the following three sources and are discussed below:

- Instrumentation limitations
- Administrative placement by NTC analysts
- Observer controller actions.

Instrumented battlefield data collected at the NTC is a record of digital data from player kits. The player kits record and transmit events such as firing, hit, near miss, kill, and use of radio. A series of fixed antennas at the NTC are used to poll (or query) the player kits every five seconds and receive the player kit data. When the player kit is polled, it reports all the events that have occurred and transmits a range pulse which is used to update the location of the player kit. The current system tracks approximately 400 player kits. [Ref. 4:pp. 13-15]

Unfortunately, the polling system used to locate players is not 100% effective. The range pulse transmitted by the player kit must be received by at least three of the fixed antennas, so that triangulation can be used to locate the player kit. If triangulation is not possible, the location of the player kit is not updated and he is considered *lost*. From 10 to 30 percent of the player kits may be lost to the computer at one time; these players are held at the last known location until updated. This is not too critical since the updates occur every five seconds. If a player kit is lost for more than ten minutes, the event data stored in the player kit will be lost. [Ref. 4:pp. 13-15]

As a result, the data recorded for any player at a given time may not be correct. A player might be shown at the last known location rather than the current location.

Unfortunately, the current data structures do not indicate which players are lost to the computer. Lost players can be seen in some battle playbacks as a player that appears to stop for a period of time and then disappears. Frequently, the player that seemingly "disappeared" did not disappear at all; he was actually moved a great distance to his correct location. Also, if a lost player is subsequently killed, and the kill is not reported, the player continues to be recorded at the last known location--alive.

The NTC analysts use administrative placement for players that are in the database, but not participating in the battle. As each battle is fought, not every player in the database is active. In these situations, the database will contain locations for the inactive players that are in administrative areas away from the battlefield. To correct for this, the data was filtered to remove players that were administratively placed away from the active part of the battlefield.

The last anomaly is due to the prerogative of the observer/controller (O/C) on the battlefield. The O/C can restore a player that has been killed, to continue the battle. This is frequently seen when a key leader is killed early in the battle. By restoring the leader, the unit can continue to fight and benefit from the training. If a player is killed in error, the O/C may restore him. Unfortunately, the time between the death and the restoration may cause that player to remain separated from his unit until after the battle has concluded.

These anomalies contribute to uncertainty when reviewing NTC battle data. The most frequent problem in the data is seen as outliers. For whatever reason, these players are recorded a great distance from their unit. Some outliers may be valid and properly recorded away from their unit. It is also possible that there were receiver-transmitter problems with the player kit that caused errors in the location database.

II. THE LOCATION AND MOVEMENT OF A UNIT

A. DEFINING THE LOCATION OF A UNIT

Historically, commanders and their staffs have used maps and symbols to visualize their forces on the battlefield. The map shows the key terrain features and the symbols represent the weapons, units and anything else that is of interest. When a symbol depicts a unit, it is a collective representation of all the soldiers and weapons assigned to that unit. Unit symbols were probably first used because the exact location of each weapon was not known, and because it was much easier to plot one symbol for each unit. Also, without representing each soldier and weapon, the map is less cluttered and the commander can more easily visualize the spatial arrangement of the units on the battlefield. This type of display is useful for military leaders not only in planning and controlling combat operations, but also in post battle analysis of combat operations.

At the NTC, the precise location¹ of each player vehicle is readily available and easily plotted using computer technology. If an aggregated unit symbol is to be placed on the map to indicate where a unit is geographically located, that symbol should be representative of the unit as a whole. The location of the unit commander or a specific weapon system would be a poor choice. When military symbols are placed on a map, the center of the symbol is meant to indicate the "general vicinity" of a unit's center of mass [Ref. 5:p. 2-26]. Clearly, some measure of the center of the unit would best suit the location for a unit symbol; however, one needs to consider operational factors to define a unit's center.

¹Locations are accurate to the limits described in Chapter I, Section E.

In a tactical sense, the center of a unit may be defined as the location where its ability to influence the battlefield--its combat potential--is centered. Since a unit uses its weapons to influence the battlefield, this would be where the unit's weapons are centered. For an armor company with 14 tanks and a few trucks, this might not appear to be a problem; but, trucks do not usually lead the tanks into battle. One would usually find the trucks a good distance away in the rear areas. Whether in realistic training or actual combat, the elements of a unit do not generally remain close together. In fact, a unit can experience a large degree of dispersion, especially during movement. Depending on the movement technique and formation, there can be a great difference in the amount of area over which a unit is spread. The dispersion of units recorded in the data used in this thesis are further confounded by anomalies in the NTC data reported in Chapter I, Section E.

In an offensive operation such as a deliberate attack, a unit will maneuver from an assembly area to an objective. Supply trucks, maintenance trucks and weapons such as tanks and armored personnel carriers (APC's) will all rally in the assembly area. At this time the unit will be resupplied with fuel, ammunition, and food. When the operation begins, the fighting forces (tanks and APC's) will move out. It would be normal to see a few elements out in front followed by the rest of the unit. Also, as the movement proceeds, the unit's lead elements may progress a great distance from the assembly area, while some last elements, "stragglers," remain. Stragglers are a common occurrence in military operations and are categorized as outliers in this thesis.

Furthermore, as a unit moves forward on the attack, it will naturally experience some linear dispersion along its axis of maneuver. This dispersion is more a characteristic of vehicle movement than it is a characteristic of attack operations. A common example occurs in convoy operations when drivers fail to maintain a fixed distance between the vehicles. This can be caused by a simple thing such as one vehicle slowing

down while driving up a hill. Those in front of the slow driver continue traveling at a faster speed, while those following are forced to slow down. In some respects this is an issue of command and control since the leader can take actions to compensate or regroup the unit. There are many other causes that are beyond a leader's control such as vehicle breakdowns, or when a vehicle becomes stuck in a ditch. Such outliers can be a great distance from the rest of the unit. Weapons, that are outliers, are not able to influence the battle if they are a long way behind. Therefore, the center of the unit should be located forward, towards its lead elements.

B. CALCULATING THE CENTER OF A UNIT

Simple measures such as the mean and median are easily applied to the grid coordinate location data of a unit. The mean center is located by averaging the X and Y coordinates separately for all the players in a unit. The median center is the median of the X and Y coordinates separately for all the players in a unit. The median center, by definition, will have an X coordinate that is in the middle of the X coordinates for the entire unit. Likewise the Y coordinate will be in the middle. Sometimes the median and the mean will coincide; but, there will always be half the values smaller than the median and half the values larger. When the mean and median centers do not coincide, the median center will always be the point closest to the majority of the weapons. This attribute of "robustness" is characteristic of the median center. The mean center, on the other hand is easily influenced by one point (outlier) that is a great distance from the rest of the unit.

Another way to gain robustness is to "trim" or remove the outlying players from the unit. When these outlying players are removed, the resulting mean center is closer to the median center. In one sense, the median center is just a mean center of a "severely trimmed" data set, since one trims away the data to find the middle point, or

two. Calculating a unit center as the mean center of a set of coordinates that has the outliers trimmed has the added complexity of the trimming procedure.

In this study three trimming methods were tested. The first method trimmed a fixed proportion of the player locations that were the greatest distance from the mean center. The second method trimmed all data points that were beyond a fixed distance from the mean center. In general, the results of these methods were somewhere between the mean and median center of the untrimmed players. Neither method showed any apparent benefit for locating the center, and they lacked the degree of robustness that the median center possessed.

The third method for trimming the player locations was documented by Joel Parker [Ref. 6:pp. 72-81] and was based upon a clustering algorithm [Ref. 7:pp. 9-27]. The clustering algorithm built clusters of the players nearest to each other and melded the nearest clusters into a new cluster, and so forth. A trimming mechanism then excluded the players or clusters that were located beyond a fixed distance from the centroid (mean center) of the biggest cluster built. This added a higher degree of computational complexity than the others. The results were almost always identical to the fixed distance trimming explained above. The differences occurred when the two methods did not trim the same players. If no players were trimmed, the result was the mean center for the untrimmed data set. Results of this method were generally located between the mean and the median centers. With no significant benefit, the computational overhead of the clustering method does not seem desirable.

C. MEASURING THE CENTER OF A REAL UNIT

Figure 4 and Figure 5 show the tank locations for two units at the NTC plotted with some measures of the unit center. Both units were actively maneuvering due West (from right to left) in a deliberate attack when these "snapshots" were taken. In both

instances, the leading elements of each unit are more closely spaced than the rest of the unit. As expected, there is dispersion along the maneuver axis for both units. This dispersion is most noticeable in Unit 1 since Unit 2 has only one trailing outlier at the far right. The "central" tanks in Unit 2, are generally more spread out than the tanks in Unit 1.

One can easily see that the median center is farther forward than the other measures of the unit center. The median center in both instances is just behind the small group of lead elements. By contrast, the mean center is the most rearward center displayed. The cluster center is a trimmed center (using the Clustering Algorithm with a one kilometer trimming criteria) lying close to the median center. The distance between the mean center and the cluster center is due to the number of players trimmed by the algorithm. As the number of players trimmed decreases, the cluster center moves closer to the mean center. If no players were trimmed by the clustering method, the cluster center coincides with the mean center; if most of the players were trimmed, the cluster center coincides with the median center.

The degree of dispersion along the maneuver axis is demonstrated by the distance between the mean and median centers in the X axis versus the Y axis. Since the movement is due West, the larger variation between the mean and median centers is in the X axis direction. There is essentially no difference in the Y axis direction.

As these two units move westward to attack the enemy, their more closely spaced leading elements will make the first contact with the enemy and have the most influence on the battle. The commander may be concerned with the stragglers; but, his primary concern is focused on the lead elements and the results of initial contact that the lead elements will have with the enemy. The median center is preferred as a measure of the center since it is forward with the concentration of weapons.

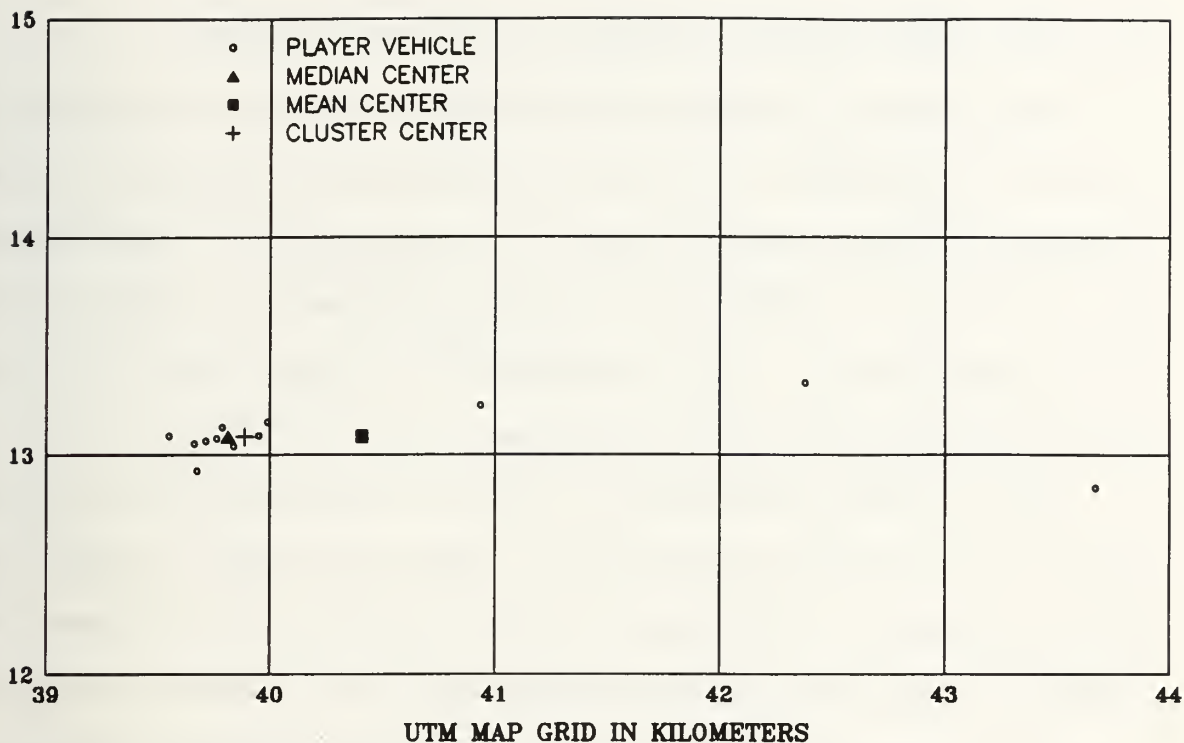


Figure 4. The Centers of Unit 1

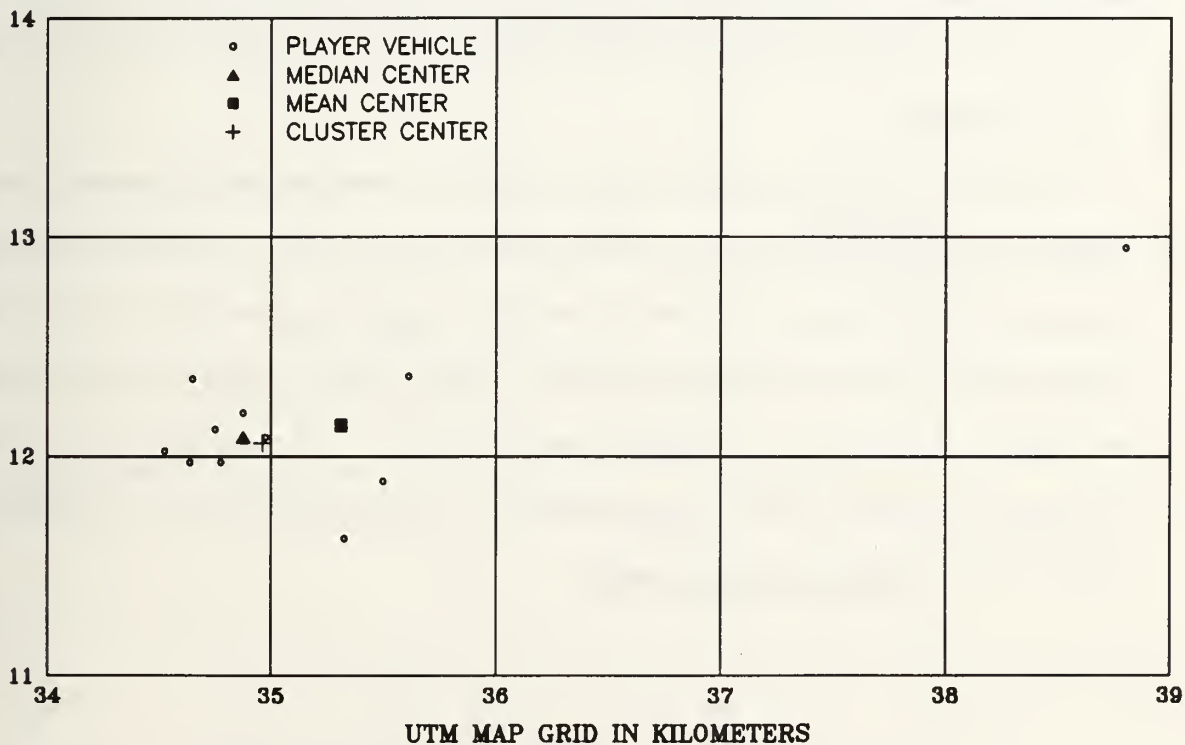


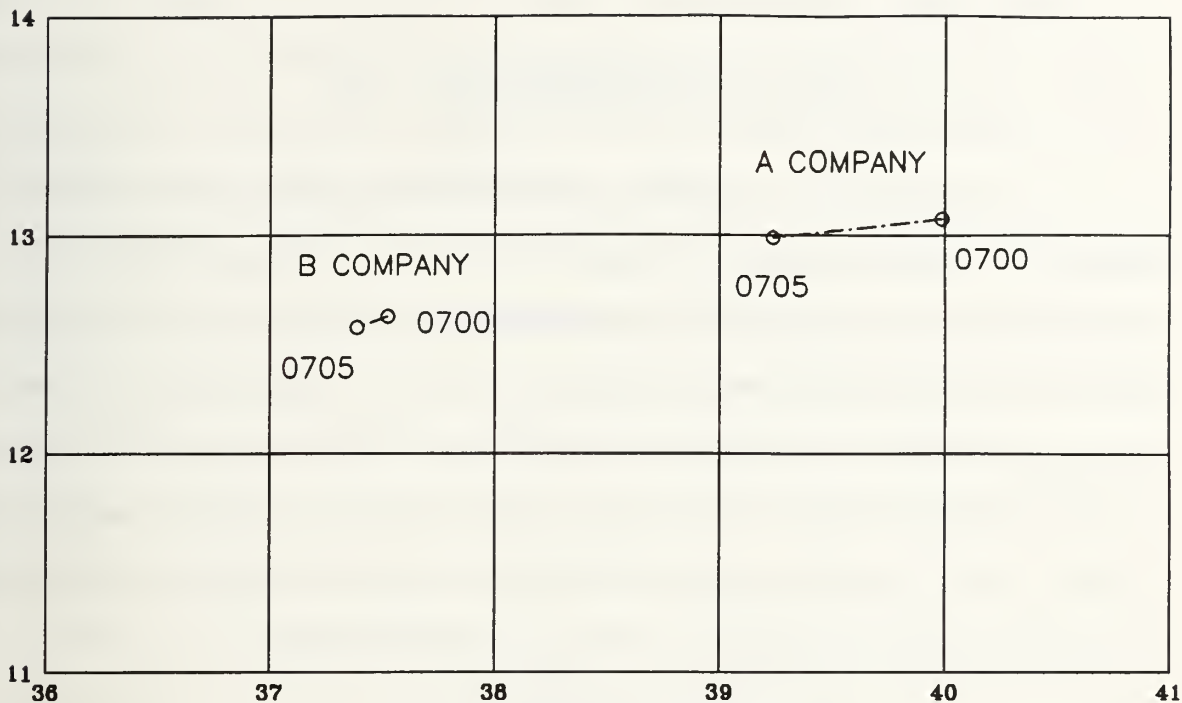
Figure 5. The Centers of Unit 2

D. USING THE CENTER TO DISPLAY A UNIT'S MANEUVER

Locating the center of the unit provides a method for examining the movement of a unit over a period of time. The NTC player location data from training battles stored at ARI-POM is normally reduced to snapshots at five minute intervals. After calculating the center of a unit at each five minute snapshot, one can plot the movement path that the unit center has taken. A visual examination of this information allows one to discern not only the path, but also the direction and the speed of movement.

Figure 6 shows the median center of two units in the same battalion at two consecutive time periods--0700 hours and 0705 hours. Both units are moving in parallel directions to the Southwest. The distance between the centers at the two different time periods is a relative indication of the speed of the unit. This is the distance the median center of the unit moved in five minutes. B Company has traveled about 200 meters and A Company has traveled about 700 meters in this five minute time interval. From this, one can readily estimate that A Company was traveling more than three times as fast as B Company.

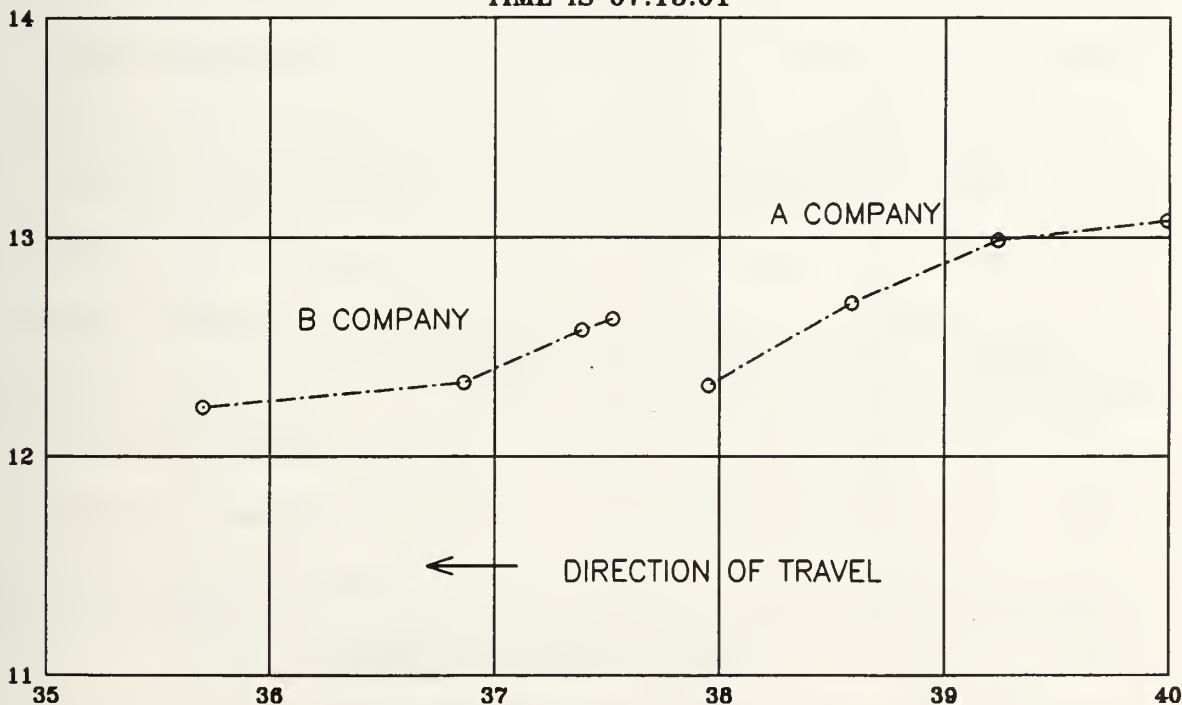
Using this type of plot one can compare the routes traversed and the relative rate of movement between the two units. Plotting the unit centers over a longer period of time shows more history of the movement in the battle. Figure 7 shows the two units from Figure 6 after ten more minutes have passed. Here, the approximate paths traveled by the units are marked with a straight dot dash line (·—·). The time is 0715 hours and the units are located at the left most unit centers. The lines help to show the changes in direction that the unit centers have made.



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Figure 6. The Movement of Two Units Over a Five Minute Period

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UTM MAP GRID IN KILOMETERS

Figure 7. The Movement of Two Units over a 15 Minute Period

Figure 8 shows the same two units, again, at 0720 hours--five minutes later. This figure is further enhanced with a standard Army symbol for an armor unit, to mark the current location of the unit [Ref. 5:p. 2-8]. An additional feature on this map display is a forward looking vector, indicated by a solid line with an arrow (→). This vector points to the center of the unit at the next time period. The forward looking vector is possible since this is a replay of a previously recorded battle. The current location of the unit is the center where the dashed and solid lines meet. Using this type of symbology to show the location of units on a computer graphic replay can present the analyst with new information about the combat operation. When the replay is stopped, the analyst can now very simply view where the unit is, where the unit was, where the unit will be, how fast the unit traveled, and the path the unit traversed.

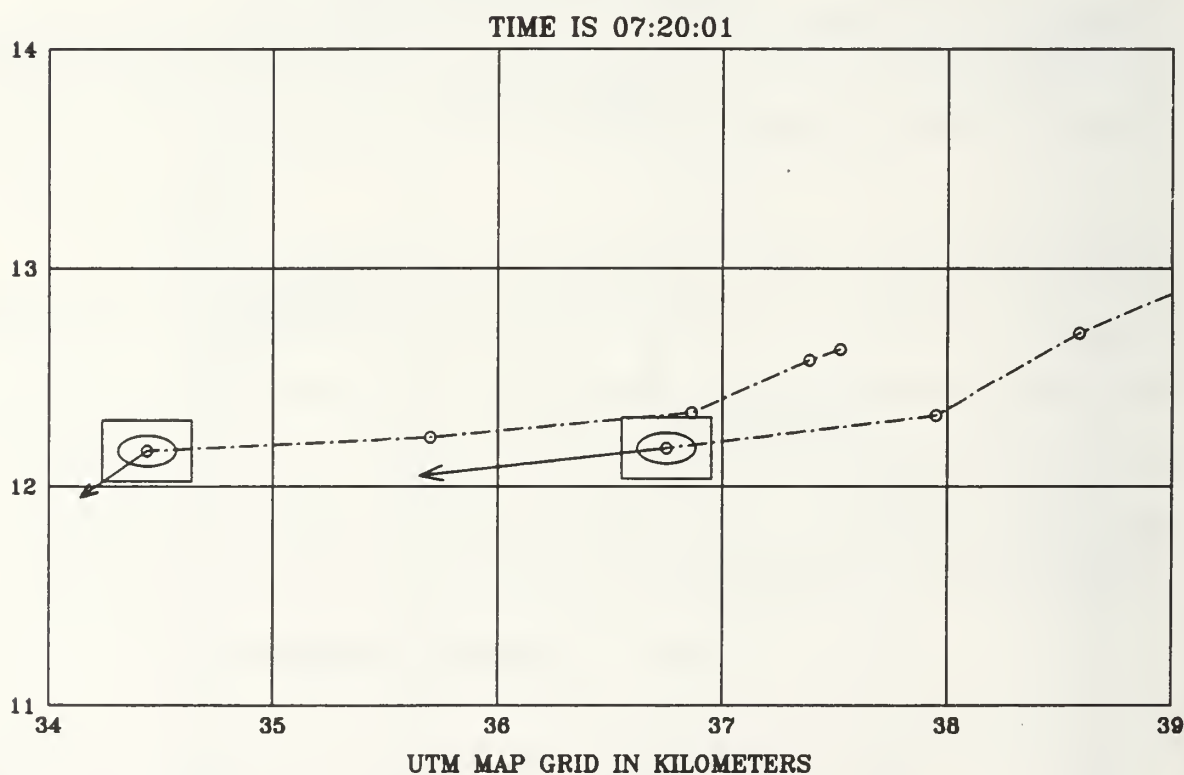


Figure 8. The Movement of Two Units on the Battlefield

III. THE DISPERSION OF A UNIT

A. DEFINING THE DISPERSION OF A UNIT

In tactical usage, dispersion describes the distance or spacing between the elements of a unit. A primary purpose of dispersion on the battlefield is protection from the enemy's firepower [Ref. 2:p. 13]. For example, typical standard operating procedures in many units require 50 meters of separation between vehicles. This is considered a minimum separation to reduce the number of casualties from the burst of a single artillery shell. Another purpose, or result, of dispersion is the effect on the concentration of firepower [Ref. 2:p. 96]. Each weapon has a limited effective fire area that is determined by its range and the direction it is pointed. As the distance between weapons increases, there is less overlap between fire areas of adjacent weapons. Consequently, the overall area of engagement of an entire unit increases, but the amount of firepower concentrated on some areas of the battlefield decreases. Less separation, or dispersion results in higher concentration of firepower on a smaller engagement area.

When planning an operation, the commander must determine the desired amount of dispersion based on the terrain, the type of threat weapons, and the type of mission (offense, defense). He adjusts the dispersion to balance survivability, firepower, and control. A unit will appear out of control when there is excess dispersion. To increase the control that a leader has over a dispersed unit, operational procedures such as formations are used during movement. Specifying a formation in a combat order is a simple means of controlling the relative positions and separation of the weapons in a unit. As the unit moves the drivers will seek to maintain their position in the formation. [Ref. 2:pp. 9-14, 62]

B. MEASURING THE DISPERSION OF A UNIT

There are many ways that one could conceivably measure the separation between weapons in a unit. The field of spatial statistics includes the study of point patterns. Figure 9 is an example of such a point pattern derived from the mapping of two units on the battlefield; the location of each tank in A Company and B Company have been marked with the letters A to K and T to Z, respectively. Some techniques in spatial statistics are based upon the distance between neighboring events [Ref. 8:p. 11,20]. These are quantitative methods that yield numerical values which would facilitate comparisons of dispersion in different units. Additionally, one could analyze dispersion using geometric methods. For example, one could calculate the area of the smallest circle encompassing the unit. In one sense, the area of that circle would reflect the dispersion of the unit. In this chapter, methods based upon inter-neighbor distances are examined. Graphical methods are discussed in Chapter IV.

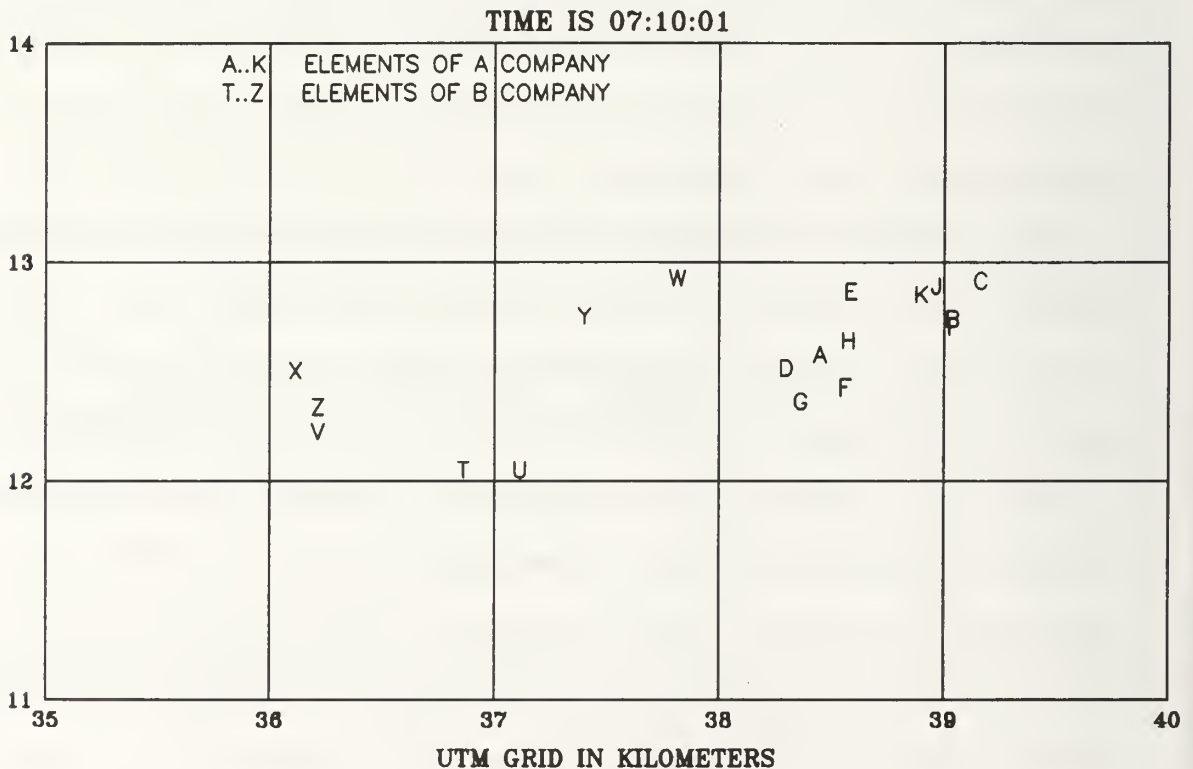


Figure 9. Mapping of Two Units on the Battlefield.

1. Using Inter-Nighbor Distances

The inter-neighbor distances (IND) are the distances between one element and all the other elements in that unit. Using a measure like IND, one could easily derive a statistic such as the mean or median IND for each element. Table 1 and Table 2 show the mean and median IND for the two units in Figure 9. The more isolated a weapon is from the rest of the unit, the larger its mean IND must be. In Table 2, the largest mean IND is 1,317 meters at location *W*, which is identified in Figure 9 as the most isolated element in B Company (approximate grid is (37.8, 12.9)).

TABLE 1

INTER-NEIGHBOR & NEAREST NEIGHBOR DISTANCES FOR A COMPANY

This table lists the various inter-neighbor distances (IND) and nearest neighbor distances (NND) in meters for A Company shown in Figure 9.

Distance Measures in Meters	UNIT AVE	LOCATION OF VEHICLE IN A COMPANY										
		B	B	E	D	E	E	G	B	J	J	K
Mean IND	446	412	423	560	524	415	432	546	362	419	421	387
Median IND	443	378	467	577	480	439	439	548	347	454	386	355
NND	132	140	40	202	162	225	188	163	140	40	74	74
2nd NND	199	162	168	215	163	312	209	209	213	195	168	178
3rd NND	225	188	178	254	277	319	213	229	225	198	198	195

TABLE 2

INTER-NEIGHBOR & NEAREST NEIGHBOR DISTANCES FOR B COMPANY

This table lists the various inter-neighbor distances (IND) and nearest neighbor distances (NND) in meters for B Company shown in Figure 9.

Distance Measures in Meters	UNIT AVE	LOCATION OF VEHICLES IN B COMPANY						
		T	U	Y	W	X	Y	Z
Mean IND	916	764	830	823	1317	901	972	803
Median IND	872	711	917	720	1292	875	882	711
NND	259	250	250	113	449	190	449	113
2nd NND	583	673	757	293	1121	293	757	190
3rd NND	866	711	917	673	1292	875	882	711

One can also summarize the unit with another statistic by averaging the mean IND for each element, which is also shown in Table 1 and Table 2. This is a value in meters that one can think of as the average distance between the elements in the unit. It is readily apparent from Figure 9 that B Company is more dispersed than A Company. Using the mean IND, we see that B Company has an average IND of 916 meters and A Company has an average IND of 446 meters. Indeed, B Company is more than twice as dispersed as A Company with this measure.

This type of measure is highly influenced by weapons that are greatly distanced from the rest of the unit. The impact of these outliers is big because each IND is used twice when calculating the unit average--once for each neighbor pair. In a unit moving down a road in a column, the average mean IND for the unit would be highly dependent upon the distance between the lead and trail vehicles. The median IND would be more robust, since the magnitudes of the largest values have no effect. The robustness of the median IND can be seen in B Company, with one isolated vehicle; the median IND is less than the mean IND. A Company does not have any outliers and its mean IND is roughly equal to its median IND.

The median IND might also have a more intuitive appeal since one can visualize the median IND at a particular weapon as the radius of a circle, centered at that weapon, which contains precisely one half of the elements of the unit. The circle analogy should not be confused with the smallest circle containing half of the elements of the unit, since the median IND circle would be centered at a weapon.

2. Using Nearest Neighbor Distances

The nearest-neighbor distance (NND) is a specific inter-neighbor distance which is a measure of the distance from one element to the nearest adjacent element. A unit summary statistic like the mean NND in a unit would be more representative of the average separation between weapons in a unit than the average IND. Tables 1 and 2

list the NND's and the unit average for the two units in Figure 9. The average separation between weapons in A Company is 132 meters. B Company is again more spread-out with 259 meters of separation. Since military commanders see the separation between weapons as a basis for defining dispersion, the NND may be a more meaningful measure than the IND.

A major limitation of the NND occurs when the elements of a unit are grouped in pairs; B Company exhibits this type of pattern. When this condition is present, the NND is not sensitive to the distance between the pairs of elements as opposed to the distance within pairs. If one were to move the three groups in B Company closer together, or farther apart, the average NND for the unit would not change. An easy check for this type of condition is to measure the second nearest neighbor distance (2ndNND). If the elements of the unit have a tendency to group in pairs, the 2ndNND would be very much greater than the NND. In Table 2, we find that the 2ndNND is more than twice the NND for B Company. The 2ndNND for A Company is only 50% greater than the NND. Similarly, if the unit was tending to cluster together in groups of three, one could examine the difference between the 2ndNND and the third nearest neighbor distances (3rdNND).

C. SUMMATION

The study of inter-neighbor distances is a worthy method for quantify the dispersion in combat units on the battlefield. The specific attributes of each measure discussed are different since they are summarizing different distances. Some method of combining several measures may provide the best means for comparative studies of dispersion and the effects on battlefield performance.

IV. GRAPHICALLY DEPICTING THE DISPERSION OF A UNIT

A. INTENT OF A GRAPHICAL REPRESENTATION

For a graphical representation, the usefulness of numerical values such as the inter-neighbor distances and the nearest neighbor distances, as discussed in Chapter III, are limited. There are many ways in which quantitative information can be graphically integrated into a display on a map. The most common methods are to use a proportionally sized symbol, a repeated symbol, or a coding method that uses color or shading patterns. G.C. Dickinson recognized in his study of statistical maps that the value of the "quantitative aspect" of data was the driving force for displaying quantitative information on maps; but he also acknowledged that...

...non-quantitative maps are often among the most successful statistical devices. Their great merit lies in their ability to *summarize* a situation. Being free from any specific code of representation they can cut away ruthlessly all confusing detail and leave only the fundamental items for the observer to notice.
[Ref. 9:p. 39]

That is the intent of this study, to develop a graphical display that is easily understood without much explanation or interpretation.

The utility of a method like proportionally sizing a symbol to display a measure of inter-neighbor distance is uncertain. If a reasonable display to show a measure of dispersion was developed, would it be more useful than simply looking at a map display of the elements in a unit (Figure 9)? The answer is probably "no" since the simple map display communicates more about the dispersion of the unit than a single numerical value. The non-quantitative map display is particularly useful for graphical analysis. For this reason, the standard methods of displaying statistical values on a map were not pursued. Another method was sought to aggregate the elements of a unit which would preserve the benefits of the simple map display.

The two-dimensional visual plot not only reveals the spacing of the elements in the unit, but also indicates how the elements are spatially arrayed on the battlefield. Returning to the basic data, the weapon location data, another method is suggested for displaying the dispersion of a unit--using a geometric figure that can be sized and oriented so that a unit's spatial arrangement on the battlefield is summarized. Dispersion information would be simply related by the relative size and orientation of the figure, and would be valid for comparison between different units. This chapter considers some methods of graphically summarizing the elements of a unit as they are positioned on the battlefield.

B. THE INTERQUARTILE RANGE BOX DISPLAY

The first graphical display developed addressed the X and Y coordinates as two separate sets of data. In Chapter II, the median X coordinate and median Y coordinate were used locate the center of the unit. For the graphical display, the variation of the data in the X direction and the variation of the data in the Y direction were used to specify the size of a rectangle. More variation in the X direction than the Y direction would yield a short, long box. If the variation in the X and Y directions were equal, the result would be a square.

The basis for measuring variation in the data was the quartile¹. The distance from the first quartile to the third quartile, the interquartile range (IQR), of the X coordinates was used to specify the width of the rectangle and the IQR of the Y coordinates was used to specify the height of the rectangle. The four corners of the rectangle were plotted at the points:

¹A quartile is division of the data into fourths. The 1st quartile divides the smallest 25% of the data from the rest of the data. The 3rd quartile separates the top 25% of the data from the lower 75% of the data. The median of the data is the 2nd quartile.

Upper Left Corner
 $X = 1^{\text{st}}$ Quartile of X coordinates
 $Y = 3^{\text{rd}}$ Quartile of Y coordinates

Upper Right Corner
 $X = 3^{\text{rd}}$ Quartile of X coordinates
 $Y = 3^{\text{rd}}$ Quartile of Y coordinates

Lower Left Corner
 $X = 1^{\text{st}}$ Quartile of X coordinates
 $Y = 1^{\text{st}}$ Quartile of Y coordinates

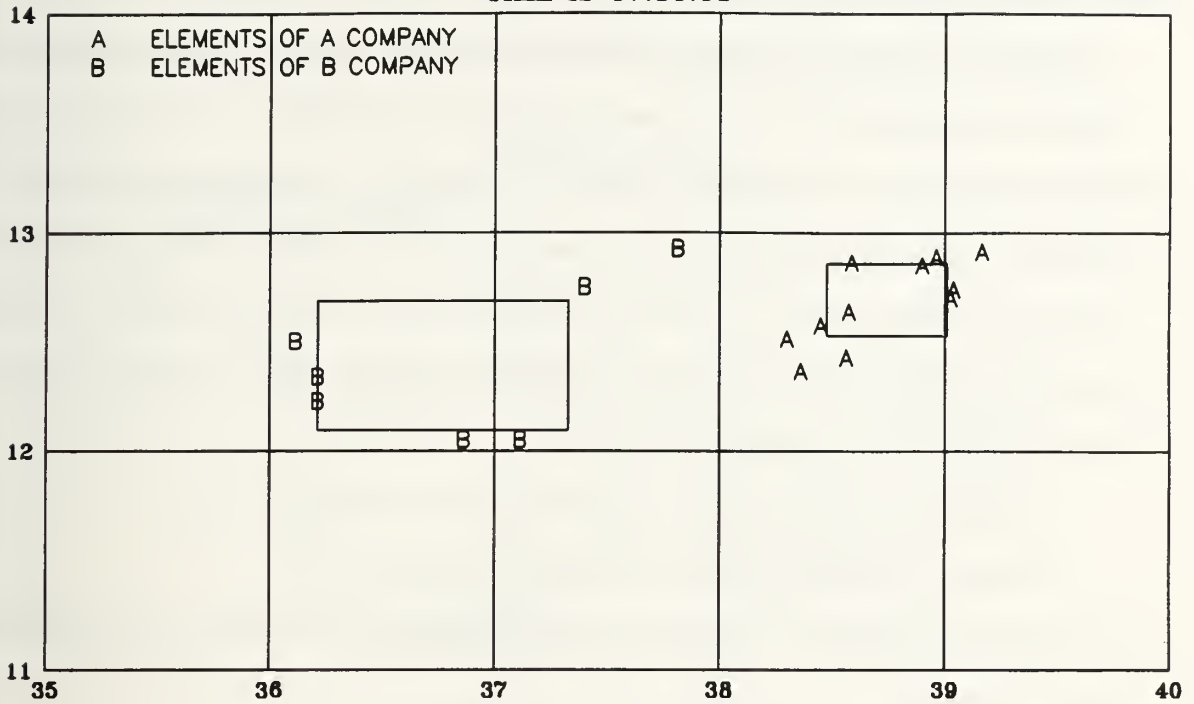
Lower Right Corner
 $X = 3^{\text{rd}}$ Quartile of X coordinates
 $Y = 1^{\text{st}}$ Quartile of Y coordinates.

With this rectangle, one could state that about one-half of the X coordinates were between the right and left sides of the "IQR Box" and that about one-half of the Y coordinates were between the top and bottom of the IQR Box. Figures 10 and 11 show the IQR Boxes for two units at two different times.

The desired conclusion was that one-half of the elements in a unit would lie inside the IQR Box. It is clear from the IQR Box representing B Company, in Figure 10, that this is not true in general. In fact, one can easily contrive a set of points such that all of the points are a great distance from the IQR Box. Still, this rectangle is a better display of a unit's dispersion than a rectangle proportionally sized to represent a value such as the unit's average inter-neighbor distance. As can be seen in Figures 10 and 11, the height and width on the IQR Boxes capture the relative dispersion of the units in two dimensions.

Unfortunately, the IQR Box's representation of the unit is somewhat misleading. The manner in which the unit was arrayed on the battlefield can be drastically different from what one would anticipate from this representation. For example, A Company in Figure 10 is roughly arrayed along a line that is rotated 45° from the horizontal. The IQR Box is limited to its vertical orientation as shown. Attempts to rotate the IQR Box were not desired since the height and width were strictly related to the vertical and horizontal axis respectively. Since there is no necessary connection between the IQR Box and the number of points inside it, this display was not pursued further.

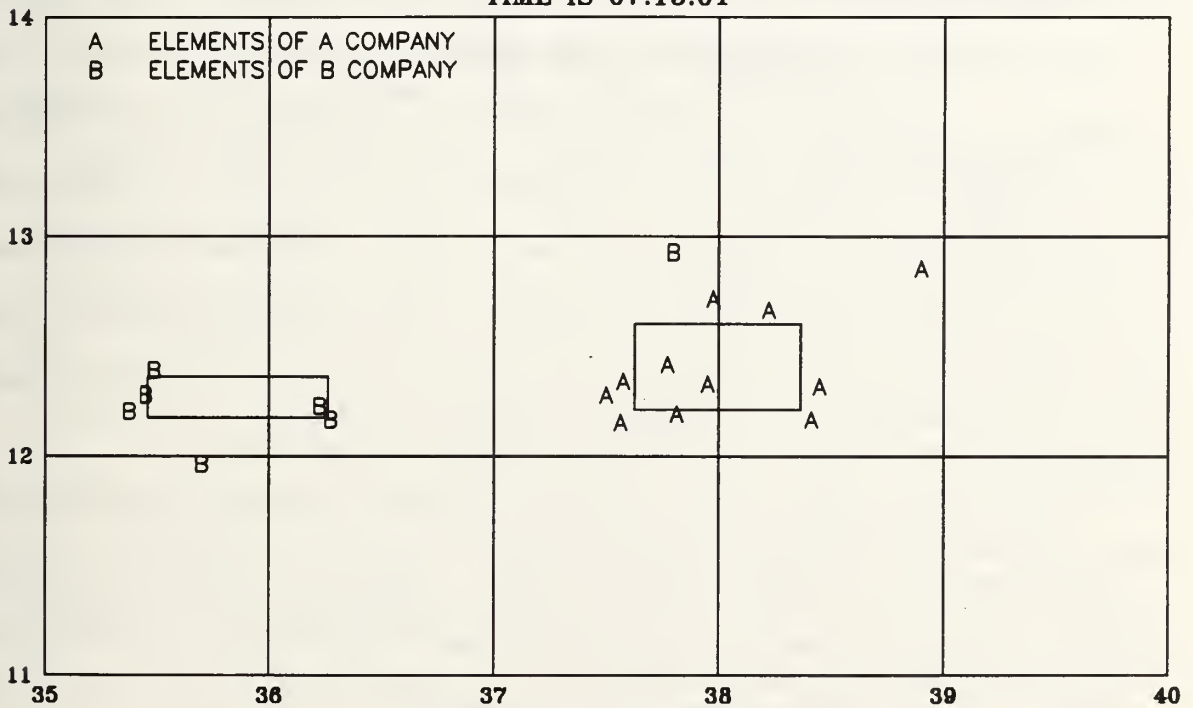
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Figure 10. The IQR Box Depiction of Two Units, Example 1

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Figure 11. The IQR Box Depiction of Two Units, Example 2

C. THE ELLIPSE DISPLAY

The second graphical display considered was more staunchly based on the actual locations on the battlefield to overcome deficiencies of the IQR Box. The objective was to draw an ellipse over the plot of weapons in the unit. The ellipse would have a height and width (length of major and minor axes), and rotation necessary to just fit around the selected points. An ellipse is defined as the set of all points in a plane whose sum distance from two fixed points (foci) is constant [Ref. 10:p. 612]. An ellipse can then be determined by three points:

- Two points to determine the long (major) axis and center
- One point that determines the perimeter of the ellipse.

The methods established to select these points determines the location, size, and rotation of the ellipse.

The location of the major axis was defined by a reference line segment between the two elements (weapons) in the unit that had the greatest inter-neighbor distance (they were the farthest distance apart). The point selected to determine the perimeter of the ellipse was the point with the greatest sum of inter-neighbor distances to the endpoints of the reference line segment. From the endpoints of the reference line segment, one can also calculate the angle of rotation for the major axis. Since these two points were always on the outer edge of the unit, outliers had a very detrimental effect on the size of the ellipse. To limit the consequences of outliers, one fourth of the unit's elements lying the farthest from the unit center were trimmed from the data set.

Several methods were used to establish the length of the ellipses' major axis; for each method tested, the width remained the same. Initially, the endpoints of the reference line segment were set as the foci of the ellipse. When defined in this manner, the major axis of the ellipse was about twice the length of the reference line segment--the

ellipse was too large. Figure 12 shows ellipses drawn around two units; the longest ellipse around each unit has the foci set at the points of the reference line segment.

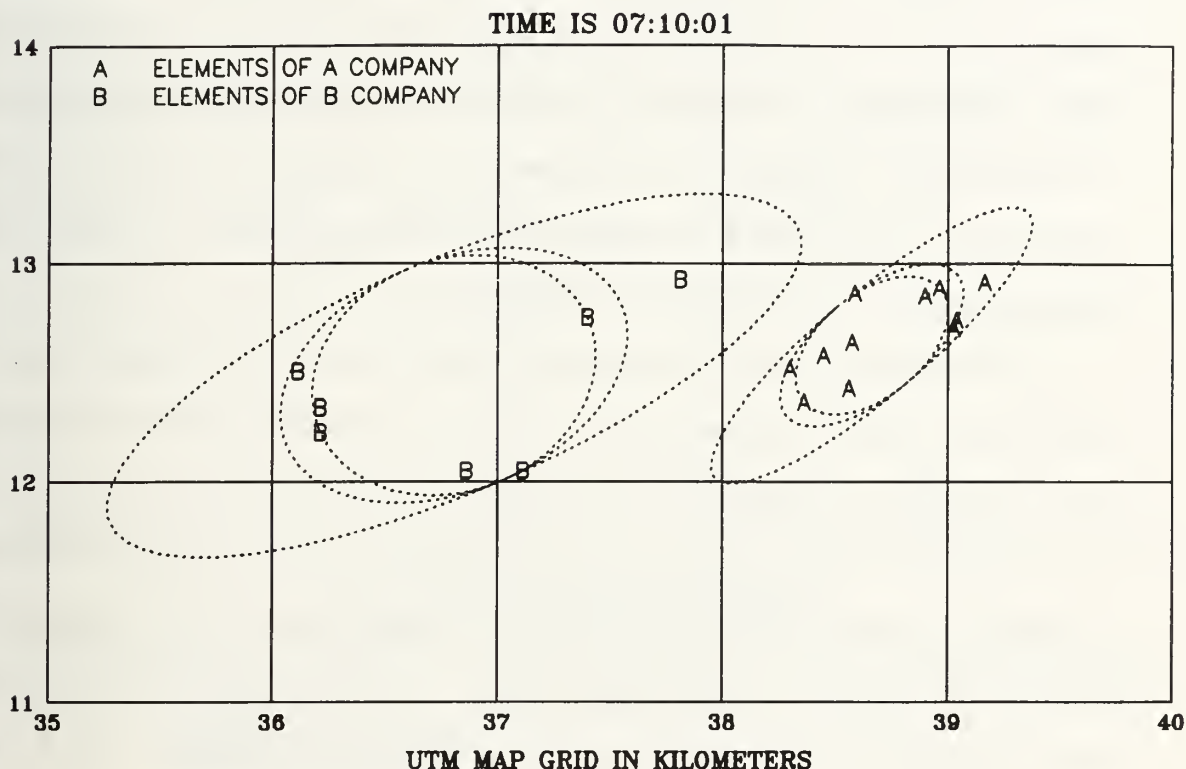


Figure 12. The Three Different Length Ellipses on Two Units

The next method used the two endpoints of the reference line segment as the endpoints of the ellipses' major axis. These ellipses are shown in Figure 12 as the shortest ellipse around each unit. Since these points were on the outer edge of the ellipse, the ellipse was much shorter and fit the unit more tightly than the larger ellipses. As a result, this method was prone to complications when the point used to define the ellipse's perimeter was beyond the limits of a valid ellipse. Generally, this point would demand the minor axis to be longer than the major axis. To overcome these difficulties, a circle was substituted in these instances. After reviewing many test plots, this method was rejected since a circle was frequently inaccurate and misleading in its representation of the unit.

Experimentation showed that when the major axis was lengthened beyond the length of the reference line segment, the complications arising from the rules of geometry were avoided. The last method tested was a compromise between the larger and smaller ellipses. This method adjusted the length of the major axis by a factor of the length of the reference line segment. This factor can be varied depending on the size of the ellipse desired. In Figure 12, the medium sized ellipse around each unit was sized as a factor of 1.25 or 125% of the length of the reference line segment. Figures 13 and 14 show examples of the ellipses on two units.

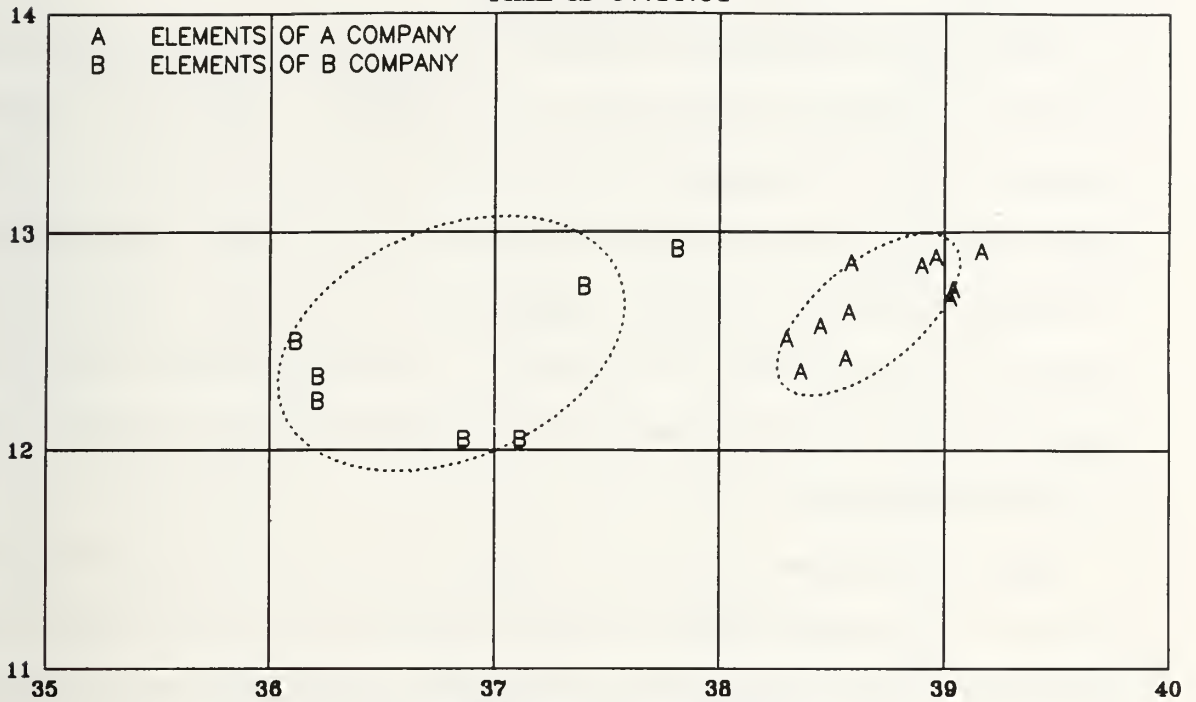
Even though the ellipse was determined from a trimmed set of data, it was frequently large enough to surround the entire unit unless there were outliers. Using this method to specify the ellipse always results with an ellipse centered at the midpoint of the reference line segment. The relative size of the ellipse was a reasonable display for comparing the dispersion between units. But, this ellipse seems to lack sensitivity to the number of elements in the unit; in Figure 14, A Company whose elements are relatively close together, has roughly the same size ellipse as B Company, which is more spread out.

Some of the other techniques tested on ellipses, that were not successful and are not shown here, were:

- Using the IQR in the X and Y directions to specify the height and width of the ellipse.
- Trimming the data set at an absolute distance such as one to two kilometers from the unit center.
- Using the extreme points in the X direction to specify the width and using the extreme points in the Y direction to specify the height of the ellipse.

While the ellipse is an esthetically pleasing shape, and shows potential for summarizing the unit, these methods of specifying the location and size were found lacking.

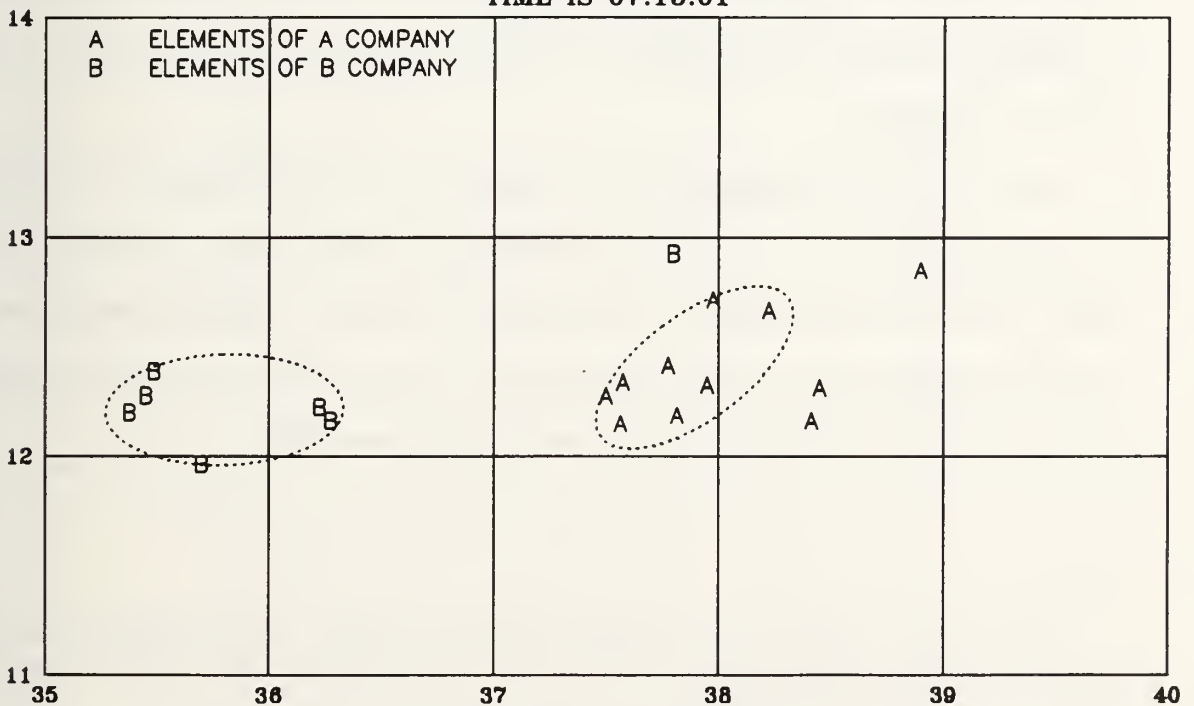
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Figure 13. The Ellipse Depiction of Two Units, Example 1

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Figure 14. The Ellipse Depiction of Two Units, Example 2

D. THE CONVEX HULL DISPLAY

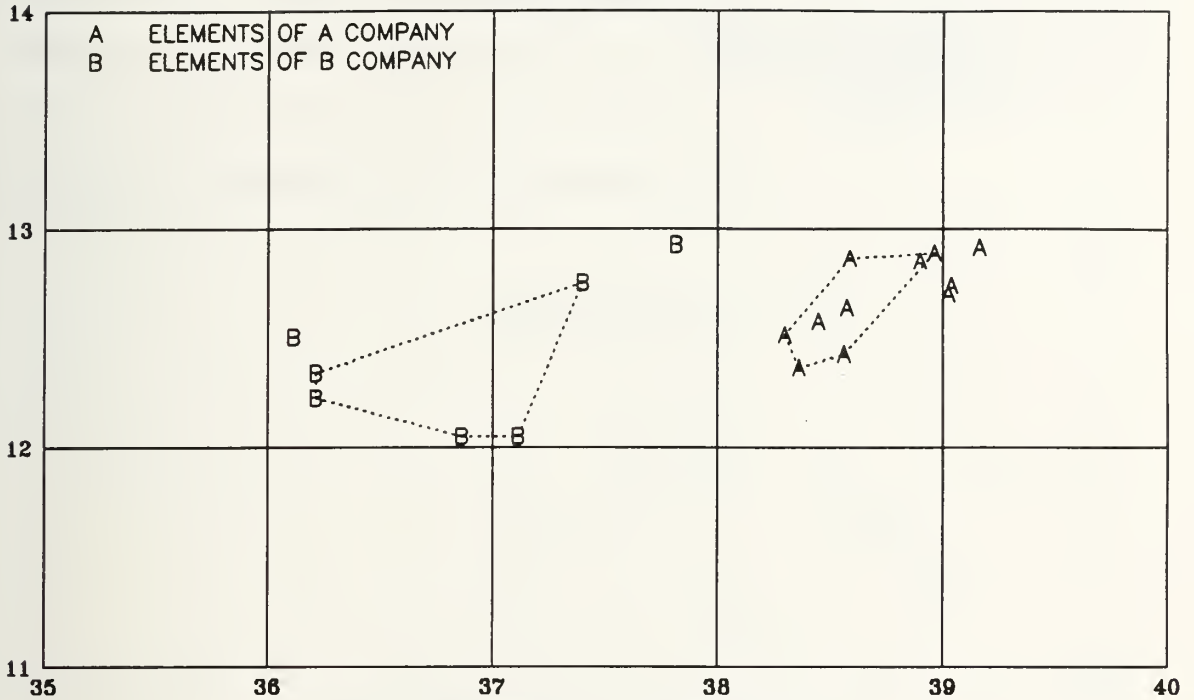
The last method considered to graphically summarize the unit on the battlefield was to draw a line around the main elements on the battlefield. This method requires a procedure to identify the points that formed the convex hull¹ around the unit. An APL function was written to implement the Graham-Scan algorithm [Ref. 11:pp. 898-902]. This algorithm conducts a "rotational sweep" from a base point to find the plotted points that form the exterior of the convex hull. To limit the effects of outliers, one fourth of the unit's elements, those that were the furthest from the unit center, were trimmed from the data set.

Figure 15 and Figure 16 show two examples of the convex hull representation of the unit. Of the graphical displays developed to represent the unit on the battlefield, the convex hull seemed to provide the most information. In comparison to the ellipse, the convex hull more accurately represents the manner in which the unit is spatially arrayed on the battlefield. Dispersion is easily inferred from the relative size and shape without any special training for interpretation. Additional examples of the convex hull are shown in Appendix A.

The convex hull is centered by the points that form the perimeter of the hull. Sometimes, as shown in Figure 17, this places the convex hull away from the unit center. This is a further indication of the dispersion in a unit. When the median center is outside of the convex hull, this indicates that elements trimmed as outliers are greatly different in location from those used to construct the convex hull.

¹To visualize a convex hull, think of the set of points as nails sticking out of a board. If a rubber band is stretched around all of the nails, the resulting shape is a convex hull. [Ref. 10:p. 898]

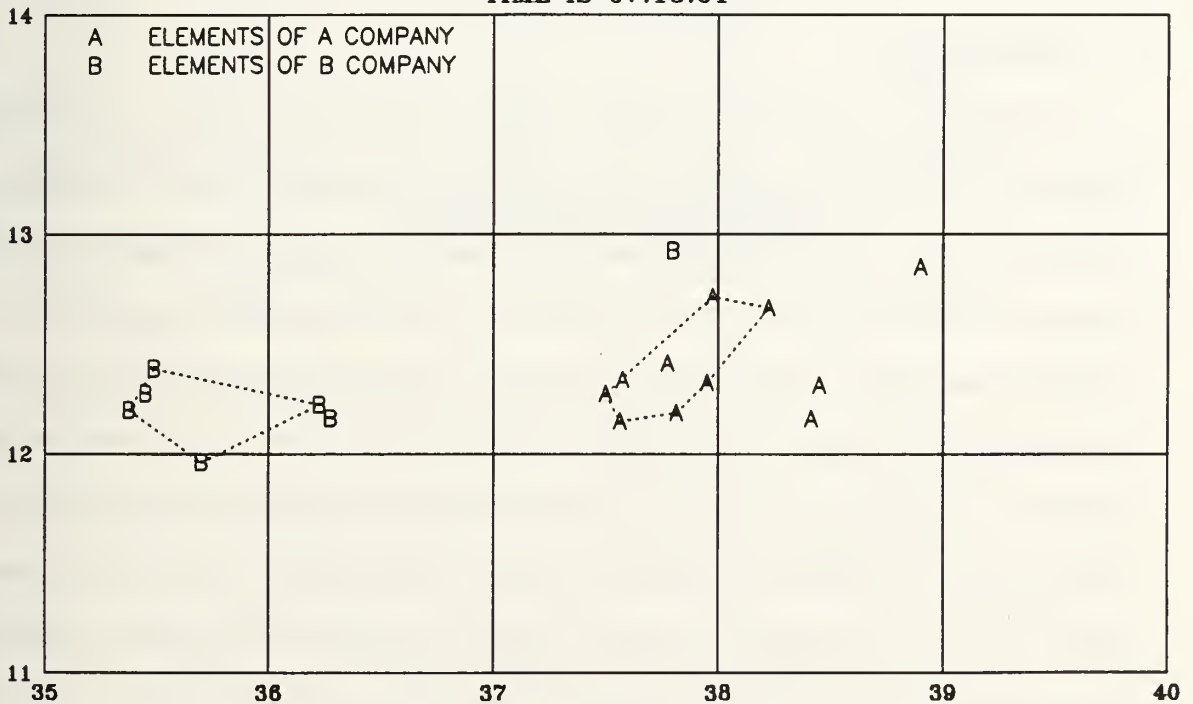
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Figure 15. The Convex Hull Depiction of Two Units, Example 1

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Figure 16. The Convex Hull Depiction of Two Units, Example 2

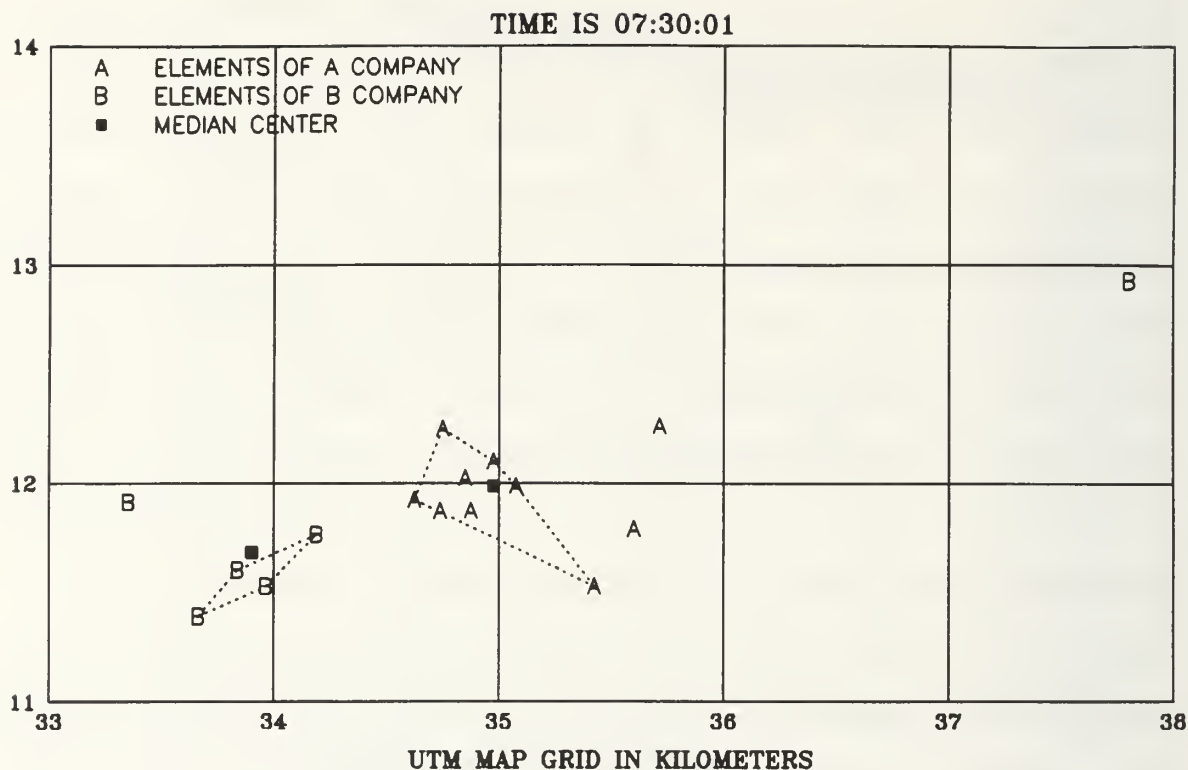


Figure 17. Example Showing the Unit Center Separated from the Convex Hull

E. SUMMATION

The desire for each shape considered was to summarize the manner in which the elements of the unit were arrayed on the battlefield and provide a basis for comparing dispersion between units. The IQR Box was not suitable since it failed to relate the position of elements in a unit. The ellipse was a suitable graphical display; but, it frequently was shaped so that it did not accurately summarize the unit. The convex hull was the most successful at depicting the manner in which the unit was arrayed on the battlefield. Figure 18 shows the convex hull and ellipse together for two units. In each case the convex hull and the ellipse were drawn with reference to the same trimmed data set. Both of the shapes provided a basis for comparing the relative dispersion between units; as can be seen in Figure 18, the convex hull is contained within the ellipse. The ellipse is liable to overstate the area the unit as it has with the unit on the

right in Figure 18. One must keep in mind that neither of these graphical displays is sensitive to the number of elements in a unit. A benefit seen in the ellipse was that it was an esthetically pleasing shape. Using a symbol such as the ellipse, would standardize the type of shape used for comparison between units.

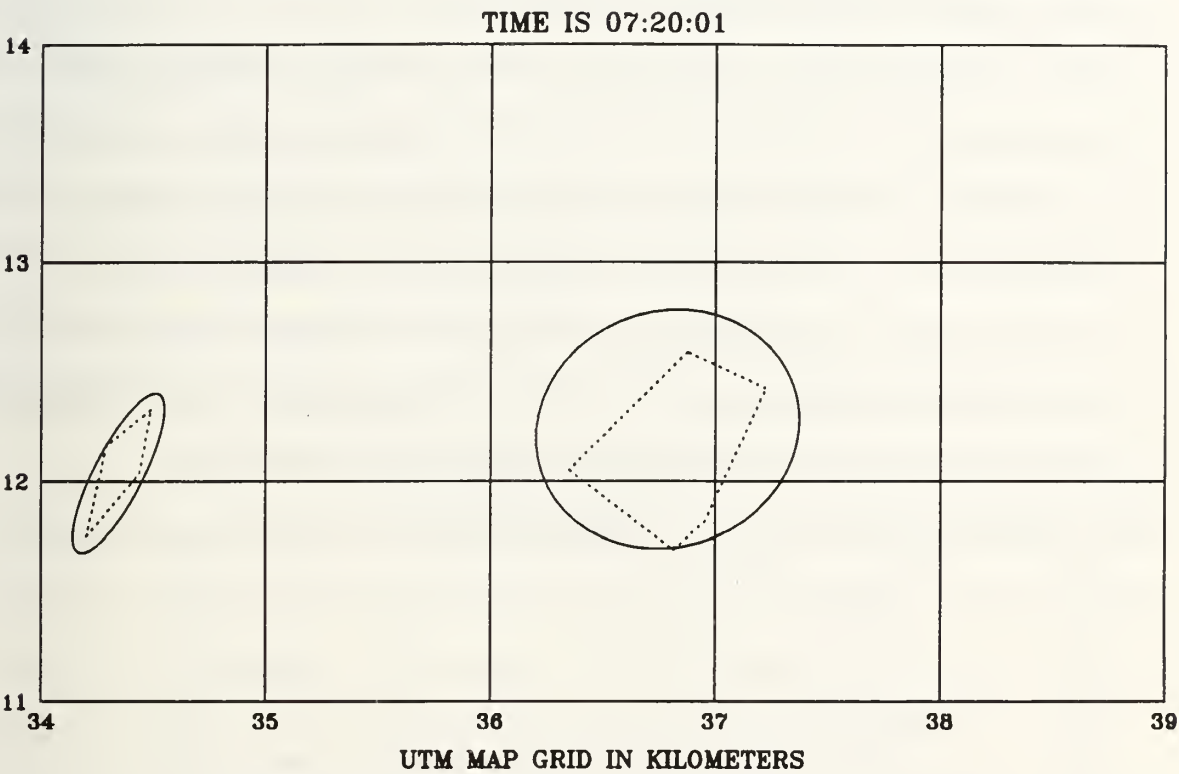


Figure 18. A Comparison of Convex Hulls with Ellipses

V. CONCLUSIONS

Analysis of battlefield training events is being conducted with a computer screen displaying a map of the battlefield. At the NTC, the computer generated map display is used to unfold the battle one piece at a time in order to see what caused the forces to achieve success or failure. Improving this current generation of battlefield displays can help remove some of the confusion and uncertainty of what is seen. For example, if one could flip a switch and replace the display of individual weapon systems, as a mass of blue squares, with aggregated unit symbols and other summary measures, then one would be cutting "away ruthlessly all confusing detail and leave only the fundamental items" on the display [Ref. 9:p. 39]. Figure 19 is an example of such a display that enables one to visualize the employment of *units* and their performance.

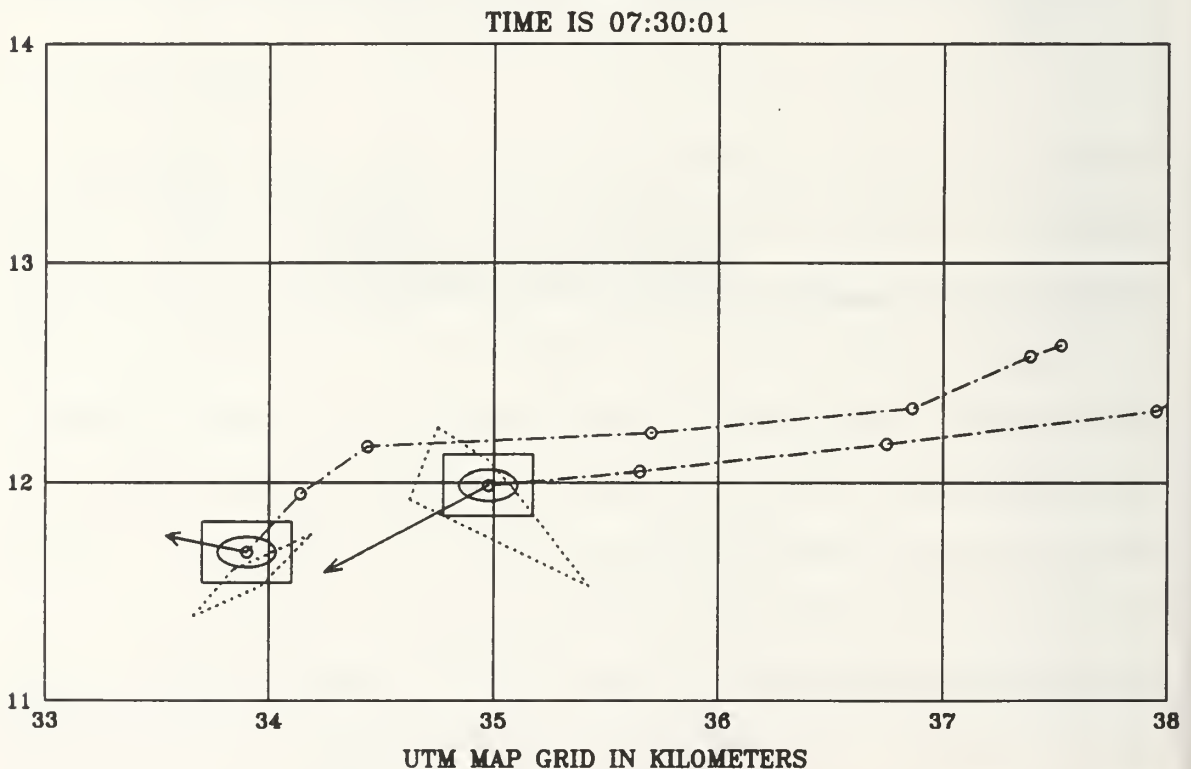


Figure 19. A Combat Maneuver Display of Two Units

The summary measures demonstrated in this thesis form a basis for developing a combat maneuver display like the one shown in Figure 19. Using these types of graphic displays, the summary of key information about units, without showing their individual weapon systems, depicts the actions of the units. These methods were demonstrated with data collected at the NTC, and provide a basis for calculating and displaying the following:

- the location of a unit
- the movement path of a unit
- displaying the speed and direction of movement of a unit
- a relative indication of the dispersion of a unit.
- the manner in which a unit is arrayed on the battlefield

Appendix A has a series of figures at five minute time intervals showing the application of these measures on two units moving to engage the enemy in a deliberate attack. Combining these elements on a terrain overlay, and given a degree of animation when replayed, the visual display can effectively describe the maneuver of the units participating in the operation. The elements of synchronization, speed and direction of movement, and the relative dispersion in units becomes readily apparent.

As research continues developing methods for improving the analysis of unit performance on the battlefield, these graphical developments are now being examined for possible implementation in the following areas:

- computer displays used in after action reviews at the NTC
- battle playback software such as GNATT and other software platforms developed at ARI-POM
- enhancements to Janus¹ as a workstation for analyzing NTC training battles.

¹A high resolution, *physics based*, ground combat computer simulation.

The Battle Enhanced Analysis Methodologies (BEAM) study continues to develop additional graphical measures for displaying other battlefield information. One such measure might be the representation of attrition experienced by a unit, or a measure of its strength. These and any additional graphics could be combined on a prototype workstation to provide an array of displays that can be toggled on-off to help analyze the many facets of battlefield performance.

The numerical measures of dispersion briefly examined in Chapter III of this study have the potential to provide valuable analytic insight into the behavior of a unit on the battlefield. For example, in Figure 20, the convex hull is used to graphically display a unit's dispersion through time. As the unit moves from right to left, it appears that the unit's dispersion is decreasing. At the far left, the unit is in heavy contact with the enemy. An in depth study of dispersion may establish its value as a useful factor in assessing unit performance.

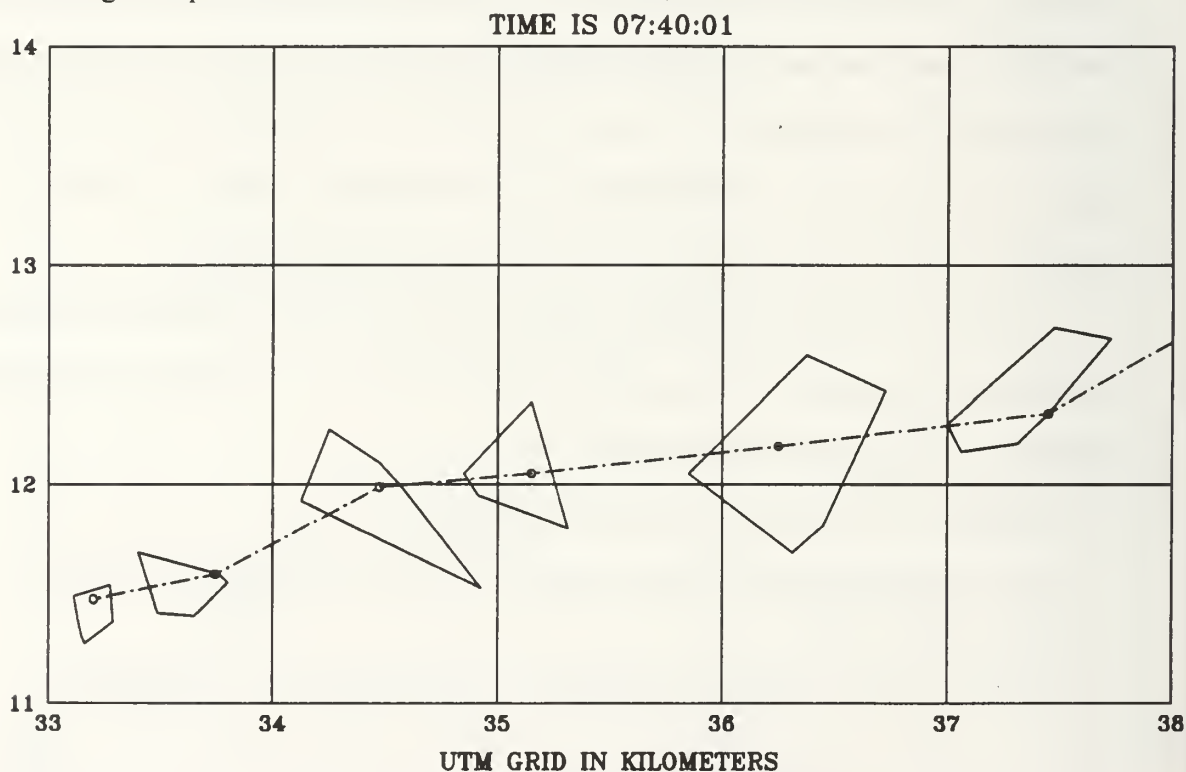


Figure 20. The Change in a Unit's Dispersion at Five Minute Time Intervals as it Moves From Right to Left to Attack the Enemy.

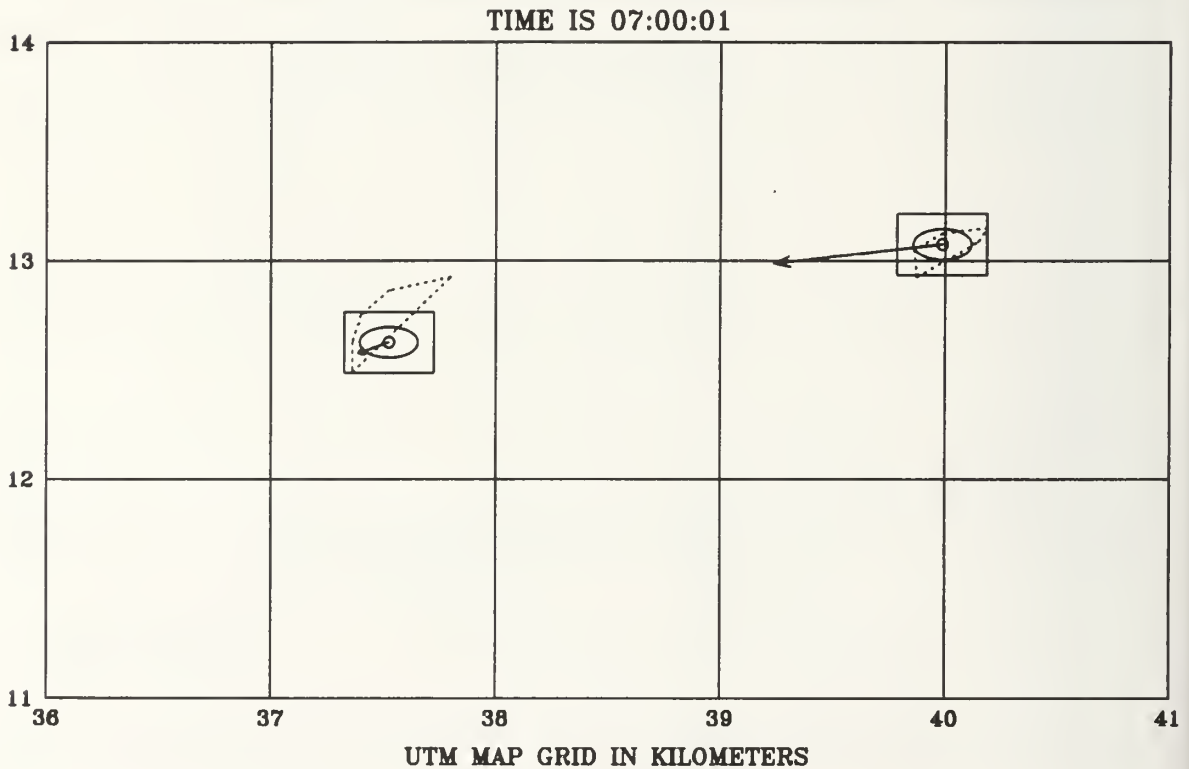
The techniques presented in this study are simple improvements for analysis that can be made to any type of map display. The map is a fundamental tool used by ground commanders for visualizing the battlefield; its role was summed-up by Lobeck in this way:

Through maps commanders make their decisions and formulate their plans. Orders are followed, units are moved into position, areas are identified, terrain advantages are recognized, and objectives are defined. Then the map continues to serve in guiding subsequent action until the mission is accomplished.
[Ref. 12: p. 1]

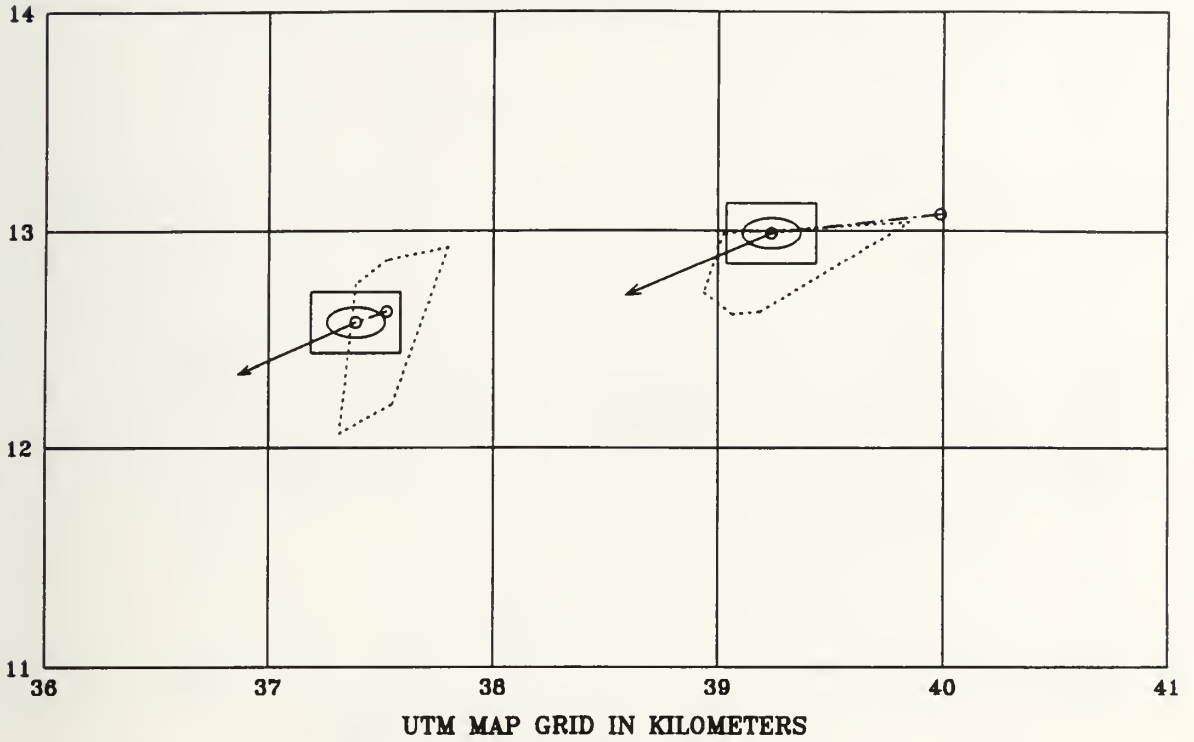
As such an essential tool for battlefield operations, a map display was a natural choice for graphically reenacting the events of a battle; and, as technology advances, the map is becoming more realistic showing increasing amounts of information. In many instances, more detail can be shown than can be assimilated. Therefore, it is necessary that new ways of displaying and summarizing information on maps be sought.

APPENDIX A: GRAPHICAL BATTLE REPLAY

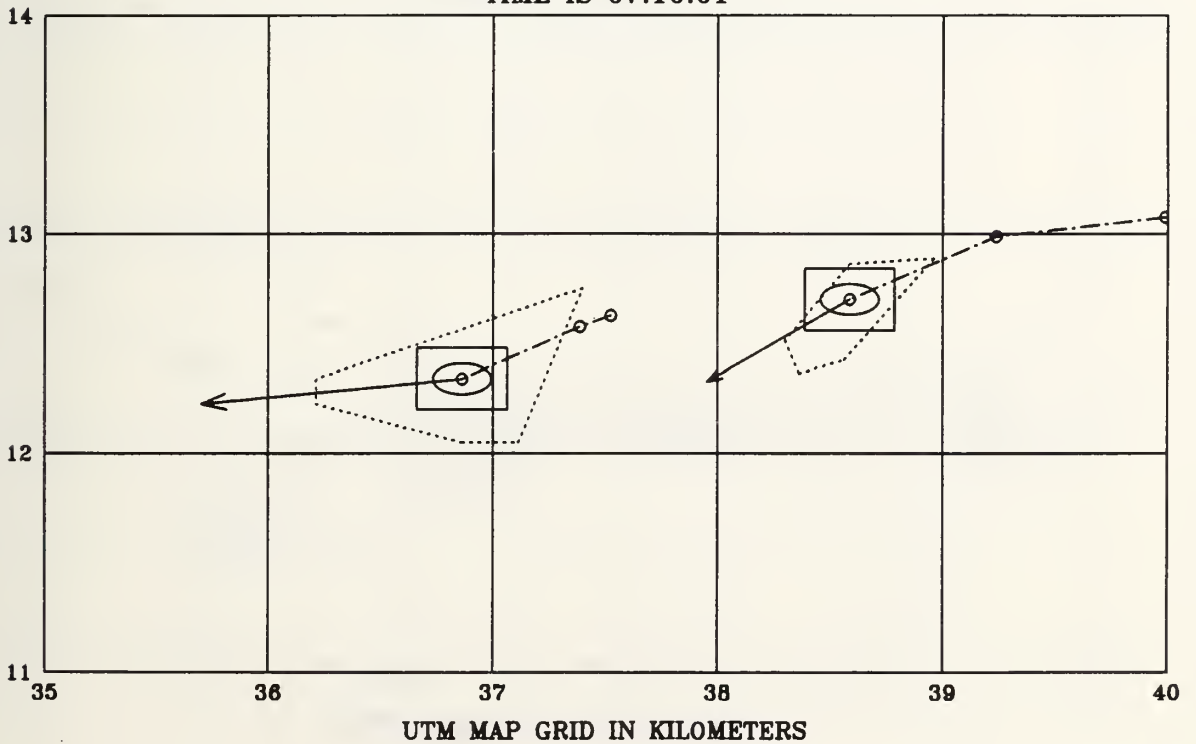
The following thirteen map displays show one hour of movement for the two units studied. The displays begin at 0700 hours and continue to 0800 hours in five minute time steps. No path history is shown at 0700 hours. The forward looking vector is not shown at time 0800 hours.



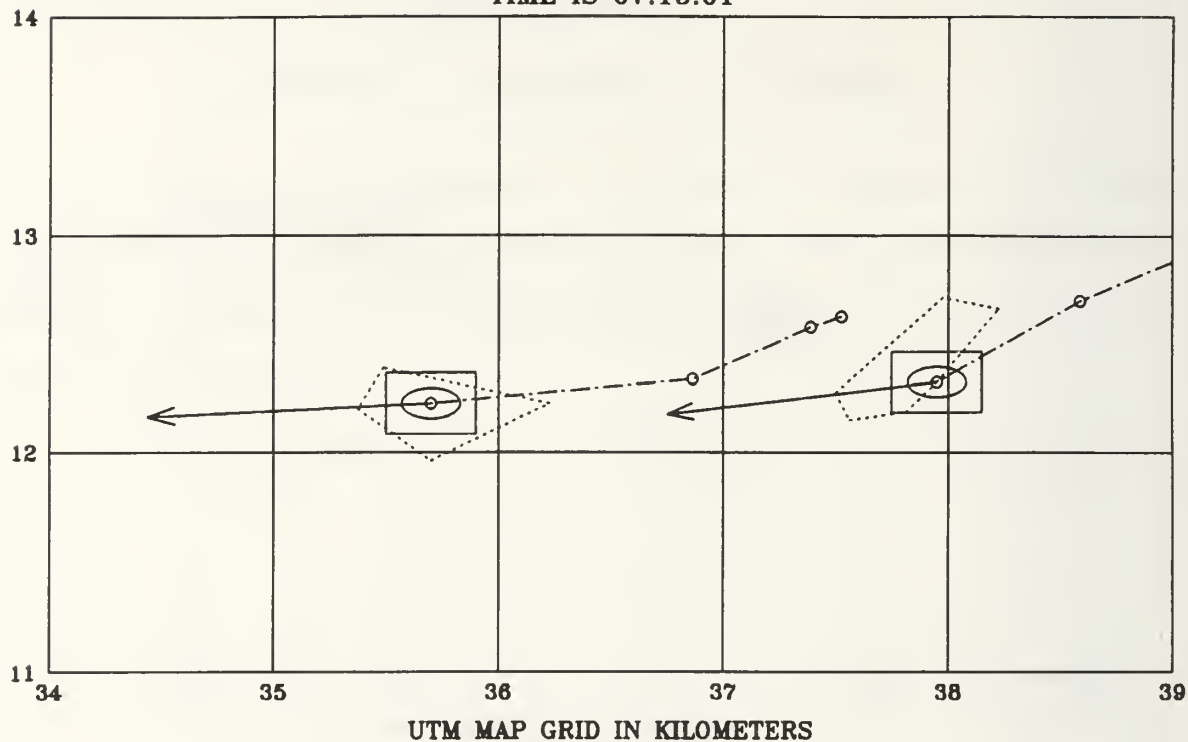
TIME IS 07:05:01



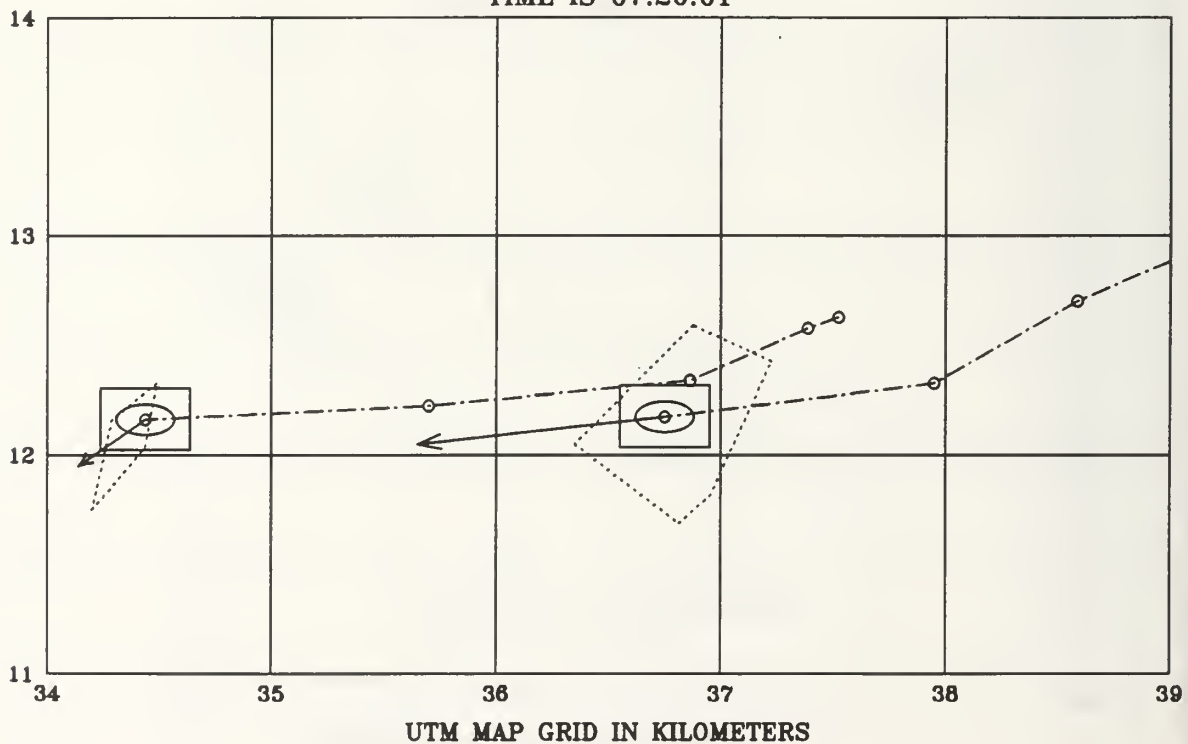
TIME IS 07:10:01



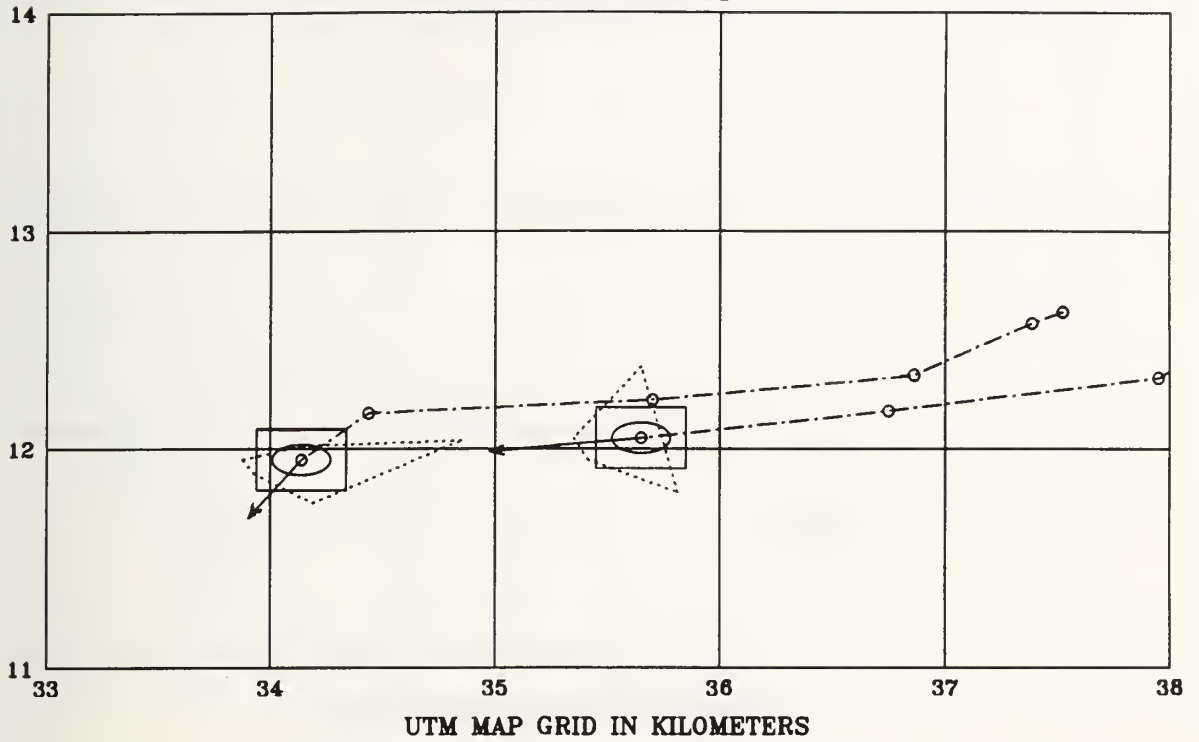
TIME IS 07:15:01



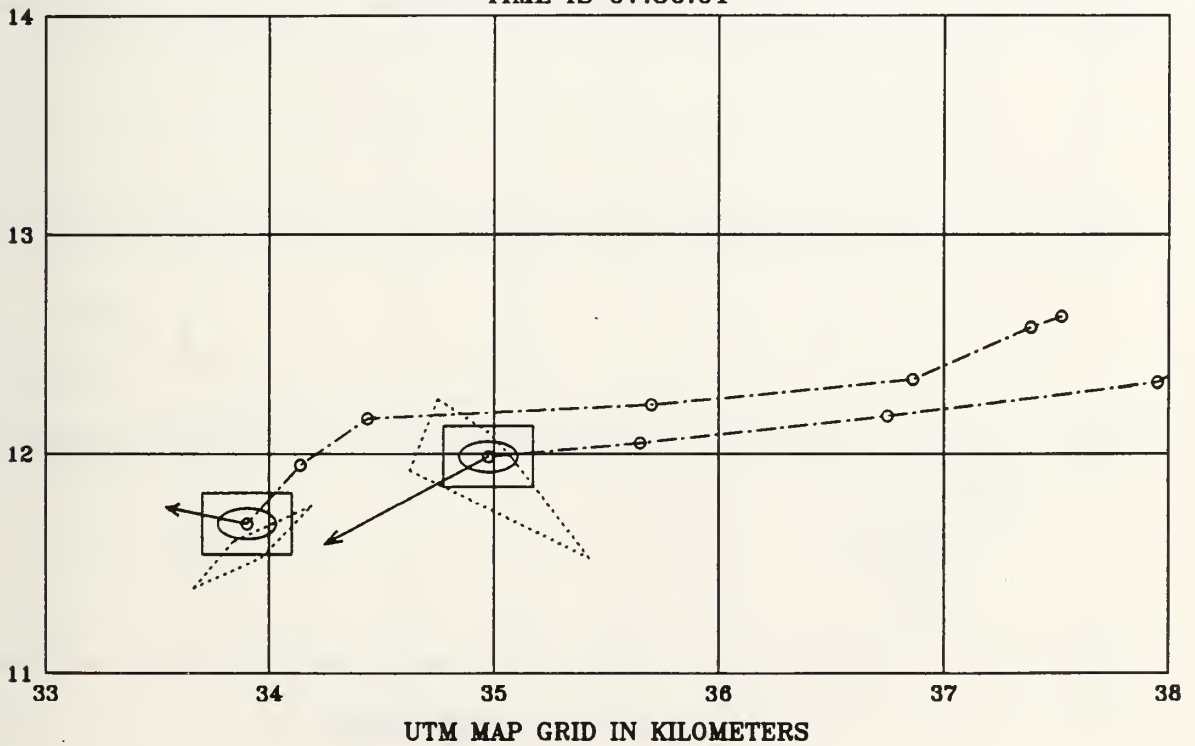
TIME IS 07:20:01



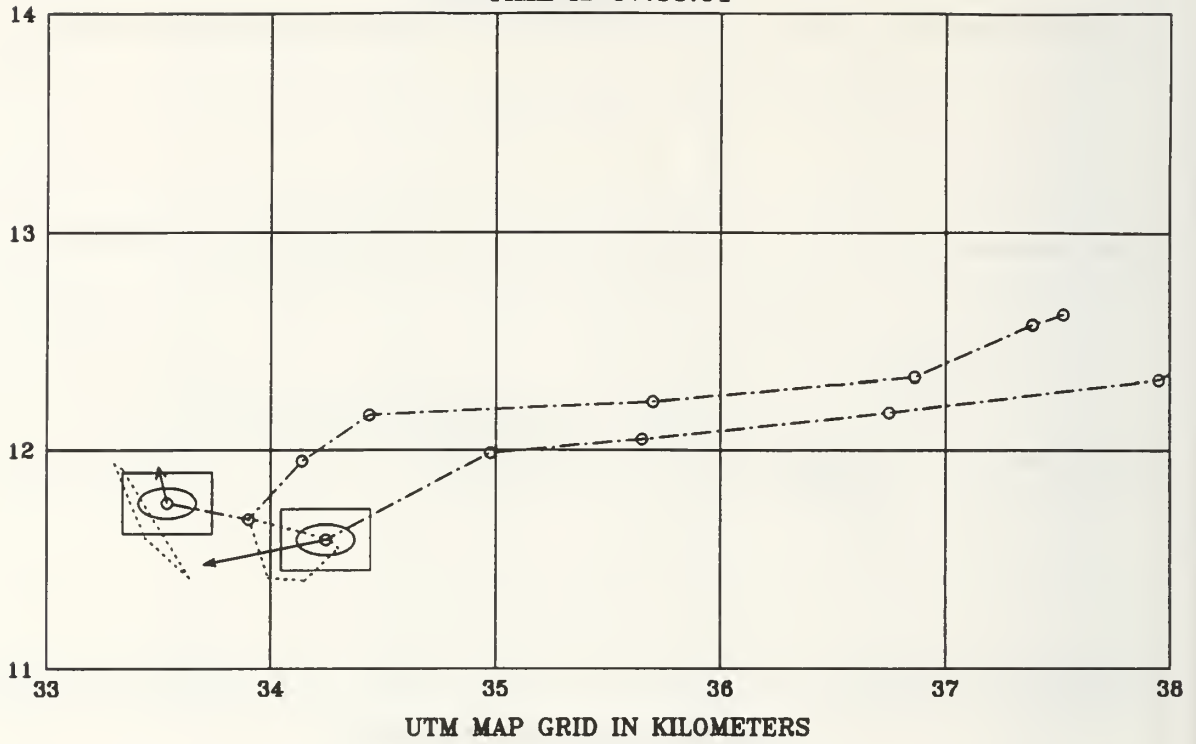
TIME IS 07:25:01



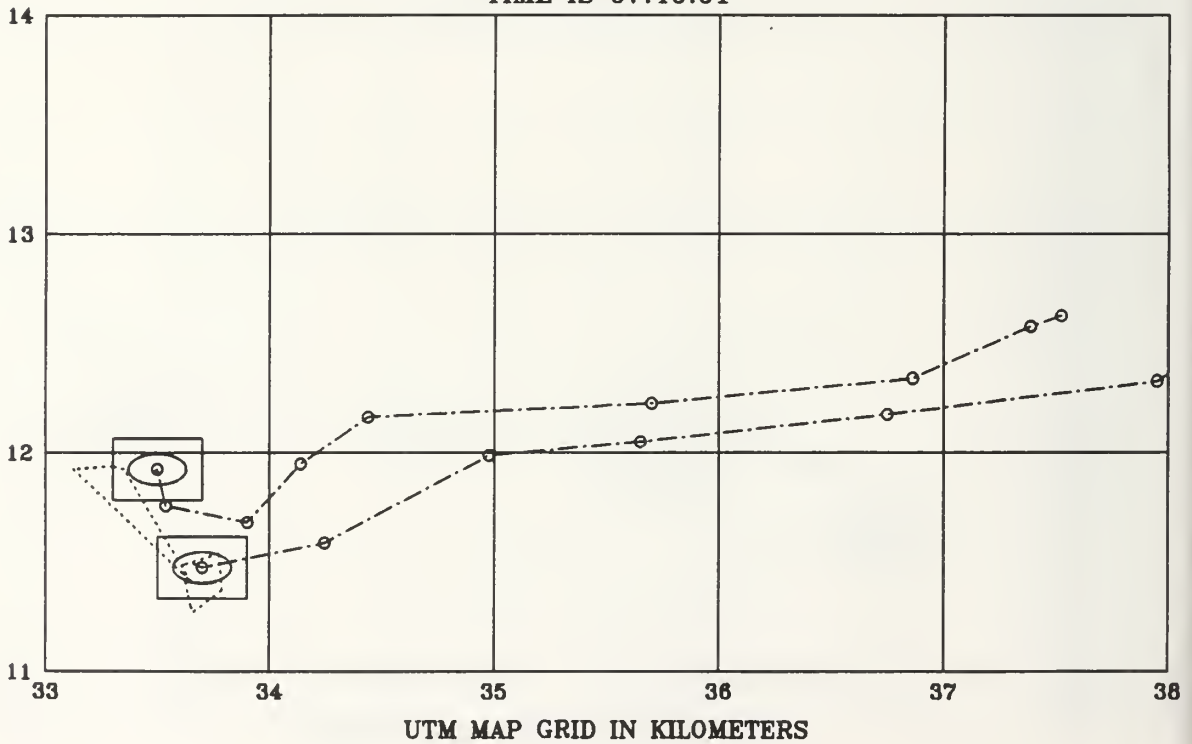
TIME IS 07:30:01



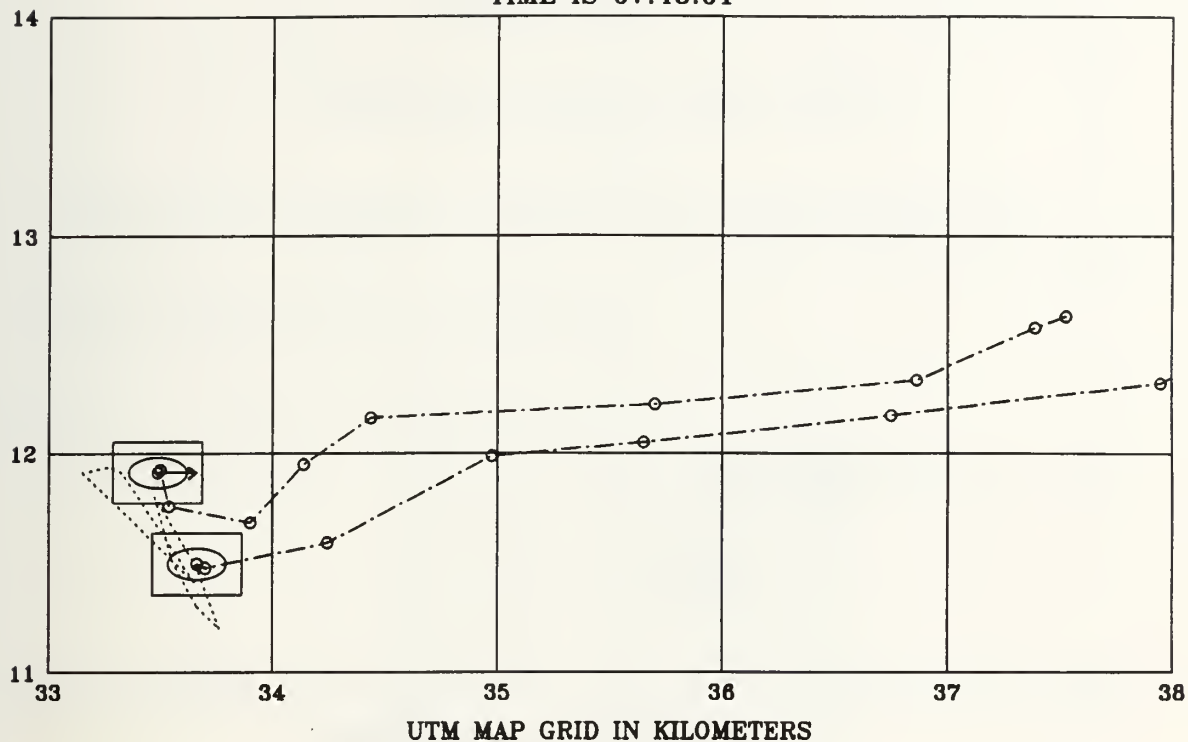
TIME IS 07:35:01



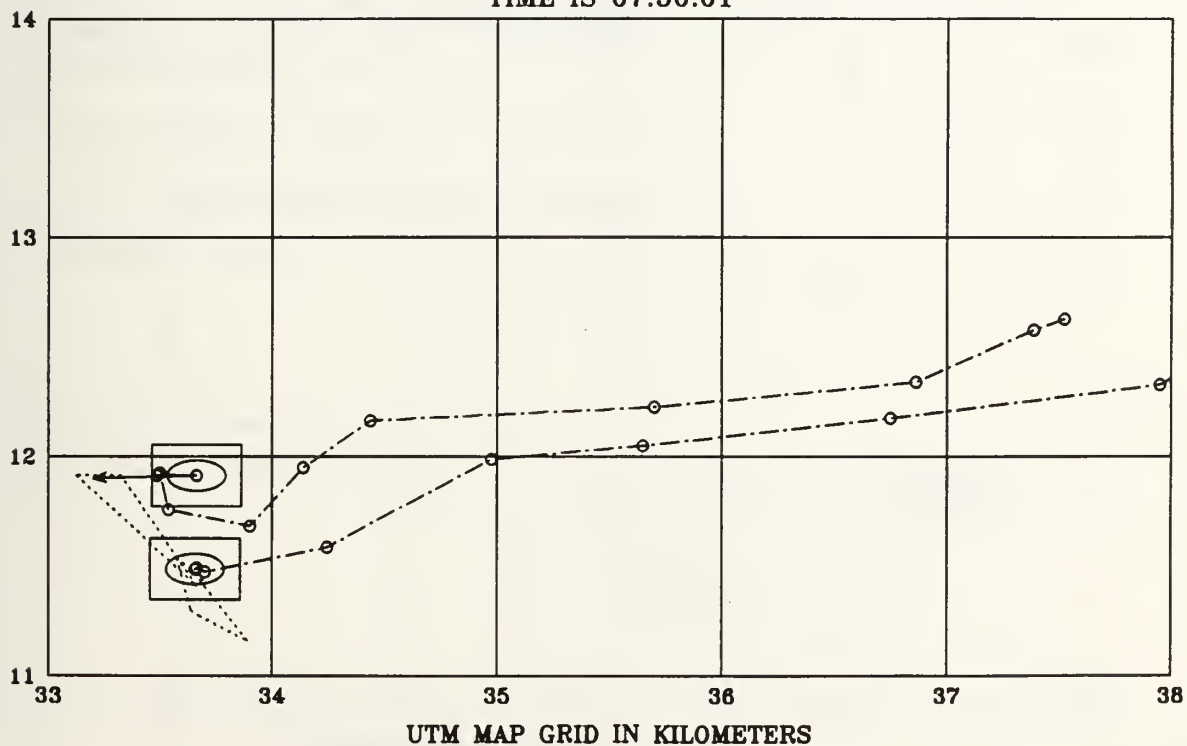
TIME IS 07:40:01



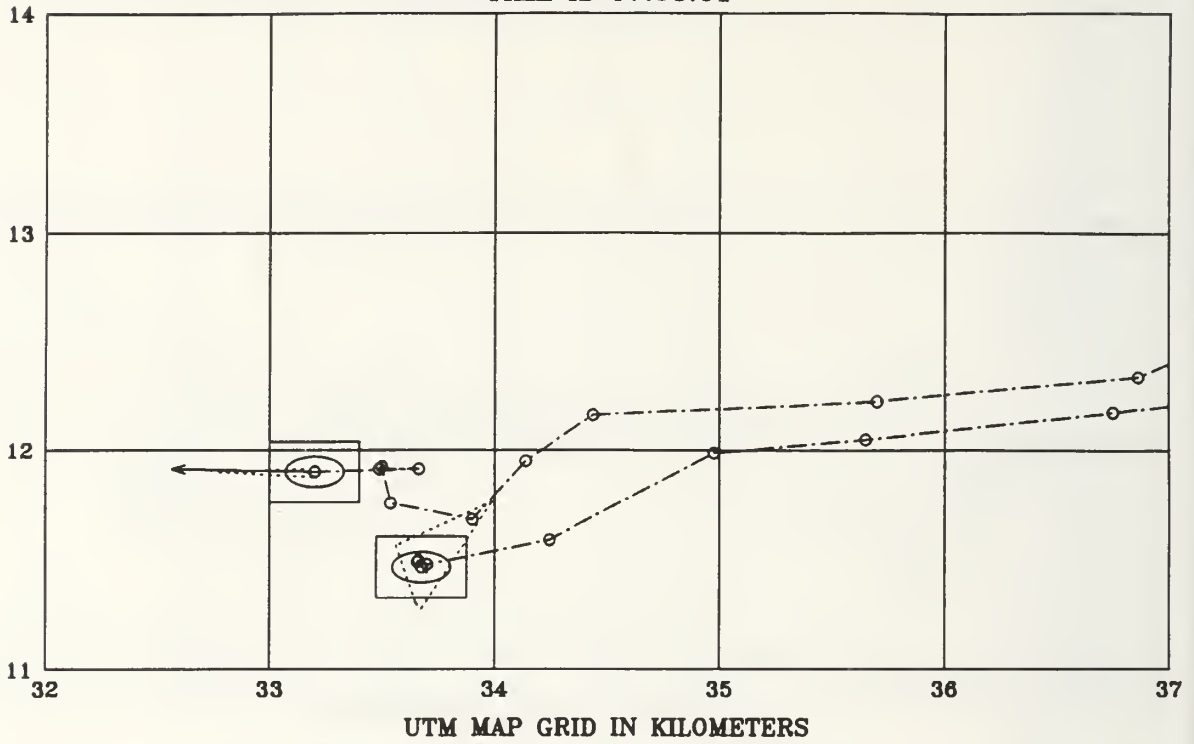
TIME IS 07:45:01



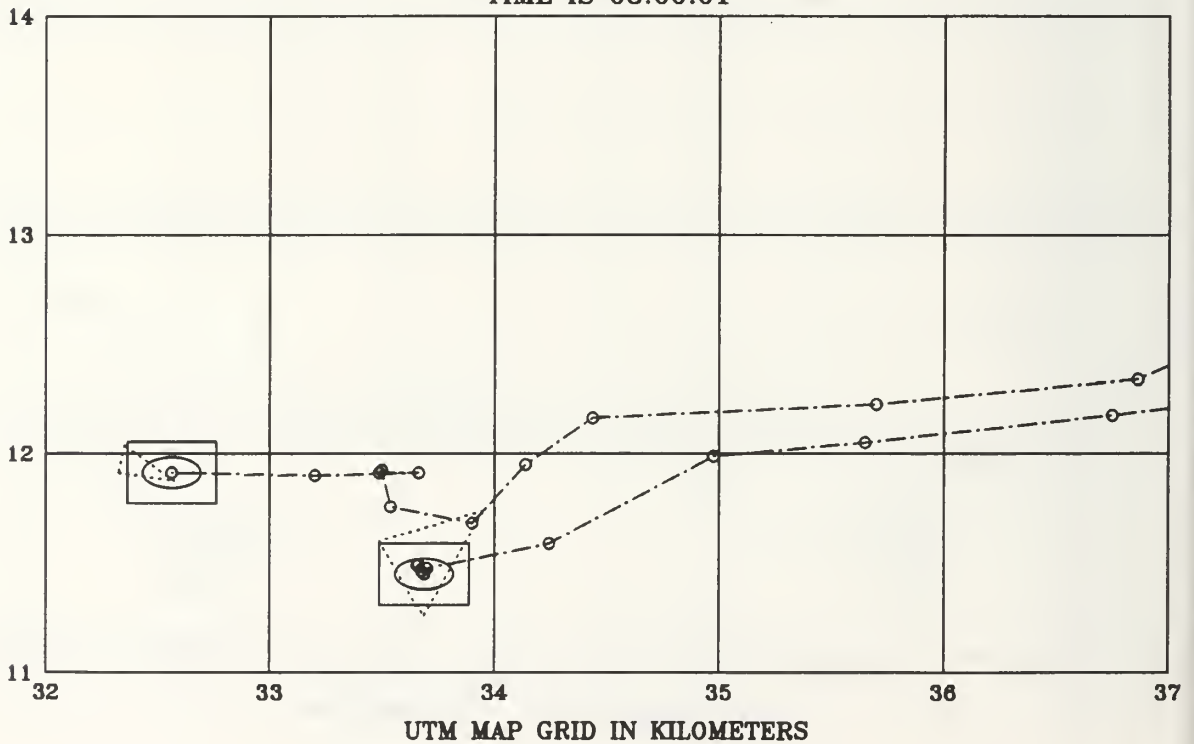
TIME IS 07:50:01



TIME IS 07:55:01



TIME IS 08:00:01



APPENDIX B: SAMPLE DATA FILES

The following are examples of the data files used in processing this study.

PLAYER LOCATION DATA FILE (PL.DAT)

```
TTTTTTTTT PPPXXXXXXXXXXXXYY  
-----  
07:27:30 1027988102000  
07:27:30 2030975114550  
07:27:30 3027988102000  
07:27:30 4027988102000  
07:27:30 5037113111675  
07:27:30 6027988102000  
07:27:30 7027988102000  
07:27:30 8027988102000  
07:27:30 9042875115700  
07:27:30 10028638102413
```

The columns of the data file are defined as:

- T Time in format of hours:minutes:seconds
- P Player numbers that range from 1 to 500
- X Grid reference coordinate in the X-direction
- Y Grid reference coordinate in the Y-direction

ORGANIZATIONAL DATA FILE (ORG.DAT)

```
PPP F SS 00000000000000000000000000000000
```

```

50 O 5 (RECON )
51 O 17 (RECON )
52 O 17 (RECON )
53 O 3 (4/3-001 )
54 O 3 (2-001 )
55 O 3 (2-001 )
56 O 3 (4/3-001 )
57 O 3 (2-001 )
58 O 3 (4/3-001 )
59 O 3 (2-001 )
60 O 3 (2-001 )

```

The columns of the data file are defined as:

- P Player numbers that range from 1 to 500
- F Friendly Force (1) or Enemy Force (2)
- S Weapon system type identification
- O Organization of the player (ASCII string in parenthesis)

MORTALITY DATA FILE (MORT.DAT)

TTTTTTTTT PPP CCCCC

07:02:02	378	DEAD
07:02:25	378	ALIVE
07:04:21	351	DEAD
07:05:02	351	ALIVE
07:05:46	383	DEAD
07:08:16	383	ALIVE
07:08:31	383	DEAD
07:09:51	383	ALIVE
07:10:00	383	DEAD
07:10:34	56	DEAD
07:11:37	383	ALIVE

The columns of the data file are defined as:

- T Time in format of hours:minutes:seconds
P Player numbers that range from 1 to 500
C Condition of player as "dead" or "alive"

APPENDIX C: AGSS/APL PROGRAM CODE

The AGSS/APL functions that are found in this appendix are the bulk of source code used in the developments of this study. Significant portions of the code are not easily communicated since they are the entries on the AGSS input screens. These elements are control vectors and are executed in the attached functions by the command `rUN`.

- *MAKEMAP* is an AGSS graphics function
- *MAKECVXH* is an APL2 function that finds the convex hull
- *MAKELL* is an APL2 function that designs an ellipse on a set of points.
- *PICKND* is an APL2 function that calculates the N^{th} nearest neighbor distance.
- *INDIST* is an APL2 function that calculates the inter-neighbor distance between points.

```

[0]    MAKEMAP;ARROWS;CENTER;HULLSIDES;NEXTCEN;TEST;UNITBOX;UNITELL
[1]    *** DO NOT MOVE OR ERASE; GRAFSTAT FUNCTION HEADER
[2]    *** GRAFSTAT WILL NOT ADD A LINE TO THIS FUNCTION WITHOUT THIS HEADER
[3]    * THIS IS AN AGSS GRAPHICS FUNCTION THAT WILL DRAW A MAP DISPLAY
[4]    * WITH UNIT CENTERS THAT CONNECTED BY A DOT-DASH LINE.  THE CURRENT
[5]    *
[6]    CENTER+2 2pCENTERA[T+1;],CENTERB[T+1;]
[7]    rUN MAPFIG    * THE BACKGROUND MAP CONTROL VECTOR FOR PLOT SCREEN
[8]    * UNITBOX IS A DATA MATRIX TO DRAW A RECTANGLE FOR STD MIL SYMBOL
[9]    * RECTANGLE IS A RESIDENT AGSS FUNCTION
[10]   UNITBOX+(2 2p0.4 0.28)RECTANGLE CENTER
[11]   rUN UNITSYMBOL
[12]   * UNITELL IS A DATA MATRIX TO DRAW AN ELLIPSE FOR ARMOR UNIT SYMBOL
[13]   * ELLIPSE IS A RESIDENT AGSS FUNCTION
[14]   UNITELL+(2 2p0.13 0.07)ELLIPSE CENTER
[15]   rUN UNITSYMBOL
[16]   * ARROW IS A RESIDENT AGSS FUNCTION
[17]   ARROWS+(CENTER MKARROWS NEXTCEN)ARROW NEXTCEN+2 2pCENTERA[T+2;],CENTERB[T+2;]
[18]   * FWDARROW IS A DATA MATRIX TO DRAW THE FORWARD LOOKING VECTOR
[19]   rUN FWDARROW
[20]   * ELLIPSEDATA IS A DATA MATRIX TO DRAW THE DISPERSION ELLIPSES
[21]   *ELLIPSEDATA+(MAKELL LA[STATA[T+1;]/INDX 11;]),[1]MAKELL LB[STATB[T+1;]/INDX 11;]
[22]   *rUN ELLIPSES    * LIGHT BLUE DOTS
[23]   * HULLSIDES IS A DATA MATRIX TO DRAW THE CONVEX HULL
[24]   HULLSIDES+MKCVXH LA[STATA[T+1;]/INDX 11;]
[25]   * LINEEP IS A RESIDENT AGSS FUNCTION
[26]   HULLSIDES+LINEEP HULLSIDES,[1]MAKECVXH LB[STATB[T+1;]/INDX 8;]
[27]   rUN CVHULLS
[28]   * PLAYERS DRAWS THE LOCATIONS OF THE ELEMENTS IN A UNIT
[29]   *rUN PLAYERS

```



```

[00] Z+MAKECVXH PTS;BP;C;CEN;CK;HULL;ID;IND;PT;R;THETA;X;Y
[01] A FUNCTION DETERMINES A CONVEX HULL FROM A SET OF POINTS.
[02] A THIS FUNCTION FIRST TRIMS 25% OF DATA POINTS THAT ARE THE
[03] A GREATEST DISTANCE FROM THE MEDIAN CENTER.
[04] A THEN PERFORMS GRAHAM-SCAN ALGORITHM TO FIND CONVEX HULL.
[05] A (SEE CORMEN, T.H., ET AL., 'INTRODUCTION TO ALGORITHMS',
[06] A MCGRAW-HILL, 1991, PP898-905) THE GENERAL ALGORITHM IS:
[07] A 1. SELECT MOST SOUTHERLY AND WESTERLY POINT AS BASE POINT.
[08] A 2. CALCULATE THE POLAR ANGLE FROM THE BASE POINT TO ALL OTHER POINTS.
[09] A 3. SPHERICAL SORT LIST...SORT THE POLAR ANGLES FROM SMALLER TO BIGGER.
[10] A 4. PLACE THE BASE POINT IS ON THE HULL LIST.
[11] A 5. CONSIDER THE FOLLOWING FOR EACH POINT ON SORT LIST:
[12] A A. CALCULATE SPERICAL ANGLE OF THE LINE BETWEEN LAST POINT ADDED
[13] A TO HULL LIST AND NEXT POINT ON THE SORT LIST
[14] A B. IF ANGLE IS GREATER THAN OR EQUAL TO PREVIOUSLY FOUND ANGLE
[15] A (START WITH ZERO) THEN MOVE POINT FROM SORT LIST TO HULL LIST
[16] A C. IF ANGLE IS LESS THAN PREVIOUS, REMOVE THE LAST POINT ADDED TO
[17] A THE HULL LIST AND GOTO STEP 5A
[18] A 6. WHEN ALL POINTS CHECKED...HULL LIST ARE POINT FORMING CONVEX HULL.
[19] A 7. OUTPUT IS AN NX4 MATRIX OF ENDPOINTS FOR EACH LINE ON HULL
[20] A COL 1 AND 2 ARE X AND Y COORD OF FIRST ENDPOINT
[21] A COL 3 AND 4 ARE X AND Y COORD OF SECOND ENDPOINT
[22] A 8. INPUT IS NX2 MATRIX SPECIFYING X AND Y COORDINATES FOR N POINTS.
[23] A COL 1 AND 2 ARE X AND Y COORD OF FIRST ENDPOINT
[24] A USE THE OUTPUT MATRIX AS INPUT FOR LINEEP (AN AGSS FUNCTION) WHICH
[25] A GENERATES ANOTHER MATRIX THAT IS INPUT FOR THE AGSS DRAW MATRIX
[26] A SCREEN WHICH WILL DRAW THE CONVEX HULL.
[27] A
[28] Q+10.75*1+PTS A TRIMMING THE DATA SET FOR ROBUSTNESS
[29] A MEDIAN IS A RESIDENT AGSS FUNCTION
[30] CEN+,MEDIAN PTS 0 IND+(CEN[1],PTS[;1])DIST CEN[2],PTS[;2]
[31] PTS+PTS[Q+Q+1+IND[1;];]
[32] A MIN IS A RESIDENT AGSS FUNCTION
[33] BP+MIN PTS[;2] 0 CK++/ID+PTS[;2]BP
[34] +(0 1=CK>1)/L0,L1
[35] L0:BP+(ID/PTS[;1]),BP 0 +L2
[36] L1:BP+(MIN ID/PTS[;1]),BP A BP HAS SMALLEST X,Y COORDINATES
[37] L2:PTS+PTS,[2]BP DIST4 PTS
[38] PTS+PTS,[2]180*(20(PTS[;1]-BP[1])PTS[;3])01
[39] A USE COSINE SINCE ALWAYS IN QUADRANTS I AND II
[40] ID+PTS[;4] A ID IS THE SPHERICAL SORT LIST
[41] A NEED TO CHECK FOR SAME ANGLES, NO NEED TO CK POINT CLOSEST TO BP
[42] ID+ID,ID[1] 0 C+1 0 HULL+1 2PID[1],0
[43] L21:+(PTS[ID[C+1];4]>0)/L3
[44] HULL+HULL,[1]ID[C+1],0 0 C+C+1 0 +L21
[45] L3:R+(((X+PTS[ID[C+1];1]-PTS[ID[C];1])*2)+(Y+PTS[ID[C+1];2]-PTS[ID[C];2])*2)*0.5
[46] +(0 1 2 3=(+/(X>0),2*(Y>0)))/L4,L5,L6,L7
[47] L4:PT+ID[C+1],180+180*(10|Y/R)01 0 +L8 A 3RD QUADRANT
[48] L5:PT+ID[C+1],360-180*(10|Y/R)01 0 +L8 A 4TH QUADRANT
[49] L6:PT+ID[C+1],180-180*(10|Y/R)01 0 +L8 A 2ND QUADRANT
[50] L7:PT+ID[C+1],180*(10|Y/R)01 0 +L8 A 1ST QUADRANT
[51] L8:+(0 1=(PT[2]<HULL[C;2]))/L9,L10
[52] L9:HULL+HULL,[1]PT
[53] +(C=1+PID)/L11 0 C+C+1 0 +L3
[54] L10:HULL+(1 0+PHULL)PHULL
[55] ID+(IDID[C])/ID 0 C+C-1 0 +L3
[56] L11:C+1+PHULL+HULL[;1]
[57] Z+PTS[HULL[1C];1 2],[2]PTS[HULL[1+1C];1 2]

```

```

[0]      Z←MAKELL PTS;CEN;DELX;DELY;F1;F2;FL;ID;IND;MAJ;MIN;THETA
[1]      A FUNCTION DETERMINES AN ELLIPSE FROM A SET OF POINTS.
[2]      A THIS FUNCTION FIRST TRIMS 25% OF DATA POINTS THAT ARE THE
[3]      A GREATEST DISTANCE FROM THE MEDIAN CENTER.
[4]      A DATA SET THAT IS GREATEST DISTANCE FORM THE CENTER.
[5]      A THEN SELECTS REFERENCE LINE SEGMENT AS THE MAJOR AXIS OF THE ELLIPSE.
[6]      A THIS SEGMENT IS DEFINED AS THE TWO POINTS THAT HAVE THE GREATEST
[7]      A DISTANCE BETWEEN THEM. THE PERIMETER OF ELLIPSE PASSES THROUGH
[8]      A THE POINT WHICH HAS THE GREATEST SUM DISTANCE FROM THE END POINTS
[9]      A OF THE REFERENCE LINE SEGMENT. THE LENGTH OF THE ELLIPSE IS 125% OF
[10]     A OF THE REFERENCE LINE SEGMENT. CENTER IS THE MIDPOINT OF REF SEG.
[11]     A INPUT IS NX2 MATRIX SPECIFYING X AND Y COORDINATES FOR N POINTS.
[12]     A      COL 1 AND 2 ARE X AND Y COORD OF FIRST ENDPOINT
[13]     A USE THE OUTPUT MATRIX AS INPUT FOR THE AGSS DRAW MATRIX
[14]     A SCREEN WHICH WILL DRAW THE ELLIPSE.
[15]     A
[16]     Q←L0.75×1+ρPTS      A TIMMING THE MOST DISTANT 25%
[17]     A MEDIAN IS A RESIDENT AGSS FUNCTION
[18]     CEN←,MEDIAN PTS
[19]     IND←(CEN[1],PTS[;1])DIST CEN[2],PTS[;2]
[20]     PTS←PTS[Q+Q+Δ1+IND[1;];]
[21]     A MAX IS A RESIDENT AGSS FUNCTION
[22]     F1←(ID←+/INDεFL+MAX MAX IND←IND[1+Q;1+Q])\1 A SELECTING REF SEGMENT
[23]     F2←F1+(F1+ID)\1
[24]     CEN←(+PTS[F1 F2;1 2])÷2
[25]     MAJ←MAX IND[F1;]+IND[F2;]
[26]     MIN←(((MAJ÷2)*2)-(FL÷2)*2)*0.5
[27]     L0→(PTS[F1;1]=PTS[F2;1])/L1
[28]     →(PTS[F1;2]=PTS[F2;2])/L2
[29]     →(0 1 2=+/(PTS[F1;2]>PTS[F2;2]),(PTS[F1;1]>PTS[F2;1]))/L3,L4,L3
[30]     L1:THETA+90
[31]     →END
[32]     L2:THETA+0
[33]     →END
[34]     L3:THETA+180×(°30(|PTS[F1;2]-PTS[F2;2])÷|PTS[F1;1]-PTS[F2;1])÷01
[35]     →END
[36]     L4:THETA+90+180×(°30(|PTS[F1;1]-PTS[F2;1])÷|PTS[F1;2]-PTS[F2;2])÷01
[37]     A ELLIPSE IS A RESIDENT AGSS FUNCTION
[38]     END:Z←((FL×1.25×0.5),MIN,THETA)ELLIPSE CEN

```

```

[0]      Z←N PICKND PTS;C;DST;ND
[1]      A CALCULATES THE NTH NEAREST NEIGHBOR DISTANCE BETWEEN EACH POINT
[2]      A LISTED IN THE NX2 MATRIX LISTING THE X COORDINATE, Y COORDINATE
[3]      A PAIR BY ROW FOR EACH OF THE N POINTS. OUTPUT IS AN N ELEMENT
[4]      A VECTOR OF THE DISTANCES BETWEEN THE iTH POINT AND NTH NEAREST
[5]      A NEIGHBORING POINT. A VALUE OF 99999 INDICATES AN ERROR.
[6]      A RESULTING DISTANCES ARE CONVERTED FROM KILOMETERS TO METERS.
[7]      N←N+1
[8]      C←1↑PTS
[9]      ND←Cp99999
[10]     DST←1000×PTS[;1]DST PTS[;2]
[11]     TOP:→(C=0)/END
[12]     ND[C]←DST[(N-1)+N+1DST[;C];C]
[13]     C←C-1
[14]     →TOP
[15]     END:Z←ND

```

```

[0]      Z←INDIST PTS
[1]      A CALCULATES THE INTERNEIGHBOR DISTANCE BETWEEN EACH POINT LISTED
[2]      A IN THE NX2 MATRIX LISTING THE X COORDINATE, Y COORDINATE PAIR BY
[3]      A ROW FOR EACH OF THE N POINTS. OUTPUT IS AN NXN MATRIX OF THE
[4]      A DISTANCES BETWEEN THE iTH AND jTH POINTS.
[5]      A RESULTING DISTANCES ARE CONVERTED FROM KILOMETERS TO METERS.
[6]      Z←1000×(((PTS[;1]°.-PTS[;1])*2)+(PTS[;2]°.-PTS[;2])*2)*0.5

```

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